Royal Norwegian Council for Scientific and Industrial Research

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by

H. Gjøystdal and E.S. Husebye

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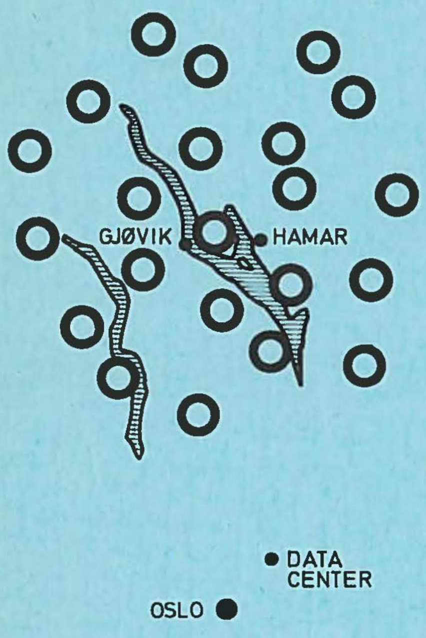

NORWEGIAN SEISMIC ARRAY

NORSAR

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This report has been reviewed and is approved.

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INTRODUCTION

The basic difficulty in detecting weak seismic waves from distant sources is to recognize the signals as such in the presence of microseismic noise. Possible solutions to this problem, i.e., to ensure the largest possible gain in the signal-to-noise ratio, would either be to suppress the noise, enhance the signal or a combination of these procedures.

The large seismic arrays LASA (Montana) and NORSAR (Norway) are mainly constructed for detecting, locating and eventually classifying small seismic events. In these recording systems, noise suppression is obtained by recursive bandpass filtering and beamforming (delay and sum processing). For more details on the NORSAR array and its operational principles, we refer to papers by Bungum et al (1971) and Ringdal et al (1972). The real-time event detector in the NORSAR software system reports a large number of weak signals, but at the same time many pure noise detections or false alarms. The problem dealt with in this paper is whether more flexible analysis techniques such as prediction error filtering (Wiener filtering), eventually combined with bandpass filters would significantly improve the SNR of detected but not verified events, as compared to an ordinary bandpass filter. The performance of the individual filters would be based on criteria like computational efficiency, filter stability and gain in SNR.

FILTER THEORY AND METHOD OF ANALYSIS

The filter in use in the event detection processor at NORSAR for additional noise suppression on the array beam level is a digital, recursive 3rd order Butterworth bandpass filter. Presently, the 3 dB corner frequencies of this filter in the array's on-line system is 1.2 and 3.2 Hz, and in average the gain in SNR amounts to around 8-16 dB depending on noise level fluctuations. The response and usefulness of a BP-filter is demonstrated in Fig 1.

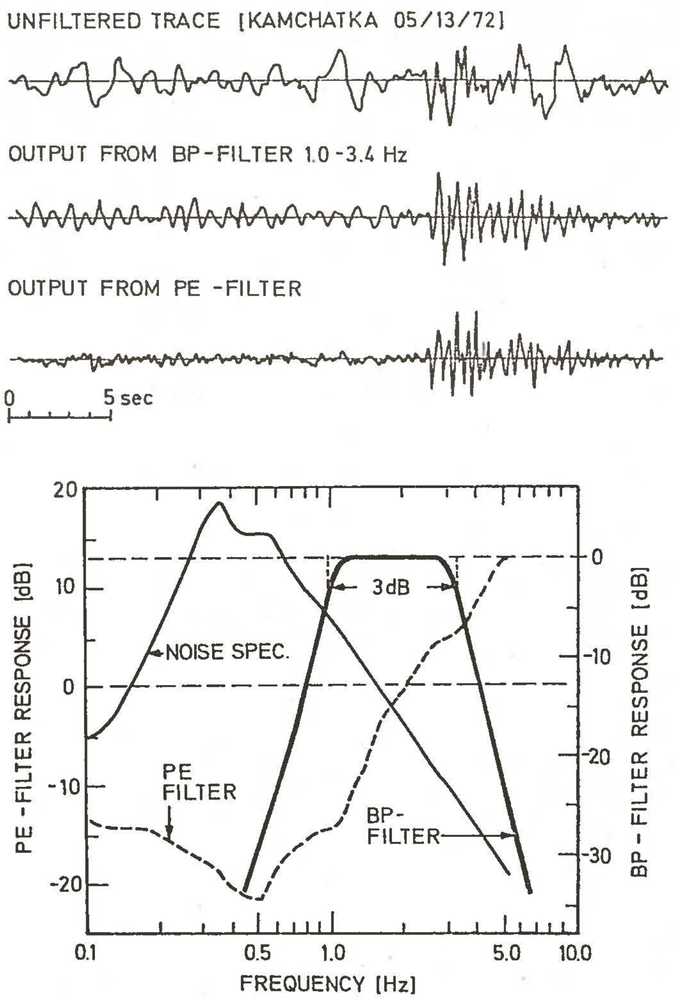

Fig 1. Examples of filtering a seismic record with BP and PE filters. The lower part of the figure shows the response of the two filters used, and gives the power gain in dB as a function of frequency. The noise power spectrum shown in the same diagram is given in dB relative to 1 nm^2/Hz at 1.0 Hz, using the left vertical scale.

Another class of filters are the so-called Wiener filters which find wide applications in seismic prospecting. The prediction error filter (PE) is a special type in the group of Wiener filters, all of which are built on the principle of minimizing the energy or power between the actual and desired filter output. This leads to a set of normal equations from which the corresponding set of filter coefficients may be calculated. In case of the PE-filter, it is constructed to predict, with the smallest possible error, a future value of the input trace on the basis of its time history. This error, i.e., the difference between the predicted and the observed value at time t, defines the output time series of the filter. In short, the PE-filter is built upon the statistical properties of the input, the output being expected to be small as far as the assumption of stationarity remains valid. Consequently, the filter tends to suppress inputs possessing the same statistical properties as the time trace from which it is constructed. Time and frequency domain response of a PE-filter is shown in Fig 1. For further details on the theory of prediction error filters, we refer to Robinson and Treitel (1967) and Douze (1971).

The aim of this work is as mentioned to investigate whether the PE-filter can be used to further improve the SNR of small teleseismic events. The requirement of computational efficiency at a large array restricts the analysis to a single time series, e.g. the array beam. Claerbout (1964) has shown that in the n-dimensional case the computer load is proportional to n^2 while the gain in SNR relative to that of a single trace was dubious.

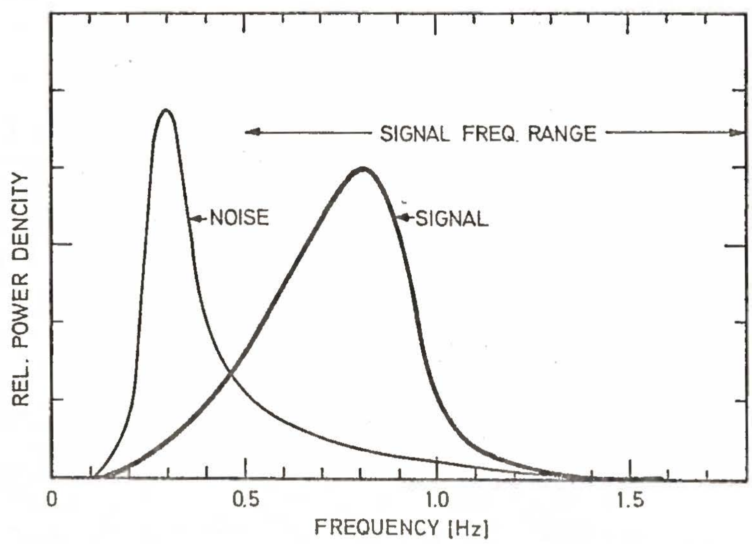

Fig 2. Models for signal and noise power spectra. The units for power density are arbitrary.

The first step in analysis was to optimize the design of the filter as its performance depends on the prediction distance α, number of filter coefficients and the length of noise interval used for autocorrelation estimation. In order to evaluate the bandpass and prediction error filter performance it was deemed preferable to work in the frequency domain and use synthesized signal and noise models (see Fig 2). The shape of the noise spectrum is rather invariant with respect to time while absolute power fluctuations exhibit large seasonal variations. To compensate for this effect the filter gain is measured relative to that of the unfiltered traces. The signal spectrum shape is also assumed to be invariant, while the peak signal frequency is varied from 0.5 to 1.8 Hz in steps of 0.1 Hz.

RESULTS

The results of the optimum design analysis of the prediction error filter are displayed in Fig 3, which shows the variation of SNR with different values of the filter parameters

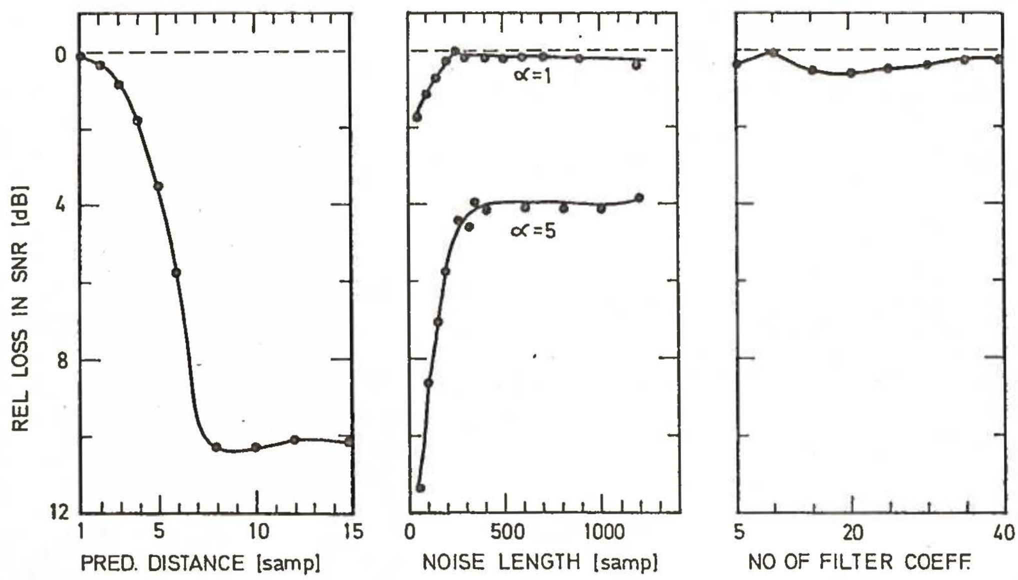

Fig 3. Loss in SNR as function of prediction distance (α), noise interval length (m), and number of filter coefficients (n). The values are in dB relative to the maximum value. Two of the parameters above are kept constant in each case; the values used are α=1 sample, m=400 samples, and n=20. In addition, the second diagram shows the graph for α=5 samples.

for an earthquake in Kamchatka 05/13/72. It is noticed that a prediction distance of 1 or 2 samples gives the best filter performance. For increasing distances α the filter performance deteriorates rapidly, which means that the PE-filter distortion of the input trace diminishes correspondingly. The length of the noise trace required for autocorrelation calculations stabilizes at 300-400 samples or 30-40 sec either a prediction distance of 1 or 5 was used. The number of filter coefficients needed is not critical as long as a minimum number limit of ca. 5 is exceeded. Analysis of other P-signals shows essentially the same picture. The effect of decreasing the number of filter coefficients is similar to smoothing the response curve.

The essential feature of the PE-filter is noise whitening, as easily seen in Fig 1. The seismic noise decreases rapidly with increasing frequency, and the filter response increases correspondingly, resulting in an increase in the high frequency noise of the output. Removal of this noise is obtainable by adding constants to the autocorrelation function at zero lag (Douze 1971), or combining the PE-filter with a suitable bandpass or lowpass filter as preferred by the authors.

From the above discussion it is clear that the PE-filter would give a satisfactory gain in SNR only for P-signals which are well separated from the noise in the frequency domain. This has been verified by analysis of the synthesized noise and signal models shown in Fig 2. The results are displayed in Fig 4 and imply that we always may define a bandpass filter having larger gain in SNR for short period P-waves as compared to the prediction error filter. Time domain analysis of real signals also proved certain BP-filters to have superior performance relative to that of PE-BP filter combinations. However, Claerbout (1964) in a similar investigation found, contrary to the authors, that the PE-filter gave larger SNR improvements than the BP-filters. The latter results seem to be due to exceptionally large signal and noise sample separation in the frequency domain.

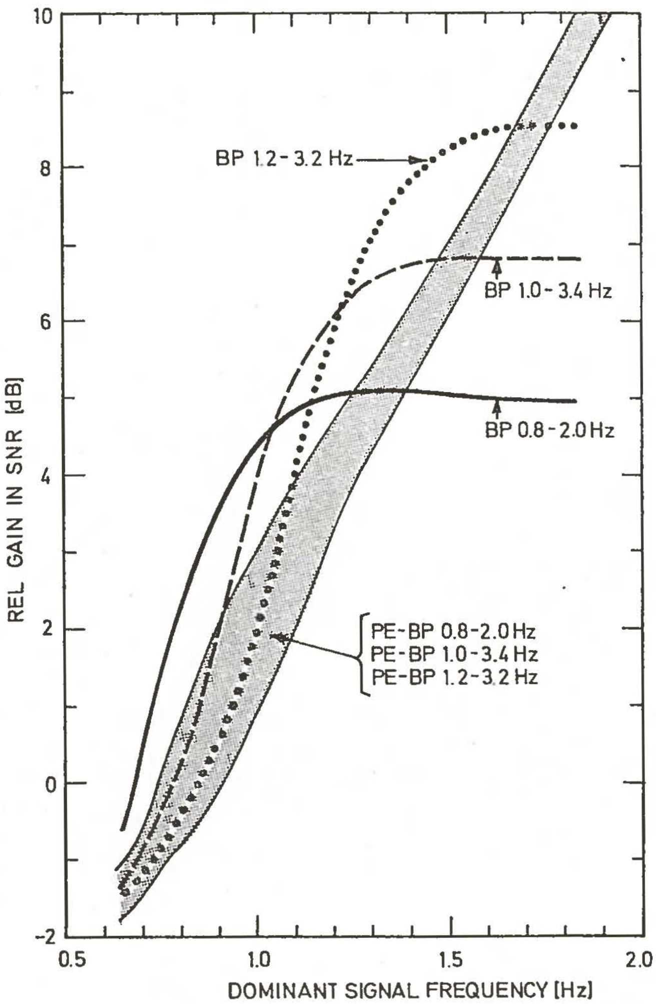

Fig 4. Gain in SNR as function of dominant signal frequency for different filters which are indicated for each graph. For simplicity the graphs for the three combination filters are shown as a shaded area, the left boundary corresponding to PE-BP 0.8-2.0 Hz.

DISCUSSION

In the previous section we dealt with filter performance in terms of SNR enhancement. For routine analysis of seismic signals the computational efficiency of the filtering process is important. It should be mentioned here that constructing and actually applying a PE-filter of 20 coefficients to a recorded P-signal required approx. 30 sec of computer time on an IBM 360/40. Without discussing in detail different types of digital seismic processing systems, we would remark that in the case of the NORSAR array, the performance of bandpass filters is superior to that of the prediction error filter for reasons of flexibility, computer efficiency and SNR enhancement.

Finally, we would like to forward a few comments on the implications of gain in SNR and thus improved seismic event detectabilit This is most easily seen through the earthquake occurrence relation (Richter 1958):

$$\log_{10}N = a-b\cdot m_b$$

where N = annual number of earthquakes in a given area, a and b are constants (b~ 1.0), and m_b = P-wave magnitude.

A certain gain in SNR (in dB) is easily converted to magnitude units (2 dB = 0.1 m_b unit), and a possible extension of the above equation is:

$$\log(N+dN) = a-b(m_b-dm_b)$$

~~$N+dN = 10^{a-b(m_b-dm_b)}$~~

Solving the above equations with respect to increments in N in per cent gives

$$\lg(N+dN) - \lg N = a - bm_b + bdm_b - a + bm_b$$

$$\frac{dN}{N} = (10^{dm_b}-1) \qquad \text{for } b=1$$

$$1+\frac{dN}{N} = 10^{bdm_b} \qquad \frac{N+dN}{N} = 10^{bdm_b} \qquad = bdm_b$$

Let us assume that the distribution of dominant P-signal period (say T in the magnitude formula) is known for earthquakes recorded at an arbitrary station. Combining this information with potential gain in SNR relative to that of routine seismometer or array operation, it would be possible to figure out the corresponding improvements in the station's event detectability. Such studies may also be regionalized if deemed necessary for special-purpose seismic surveillance.

$dm_b = 0.1$ ¢ giv $\frac{dN}{N} = 10^{0.1} - 1 = 0.26$

$dN = 0.26 \cdot N =$

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