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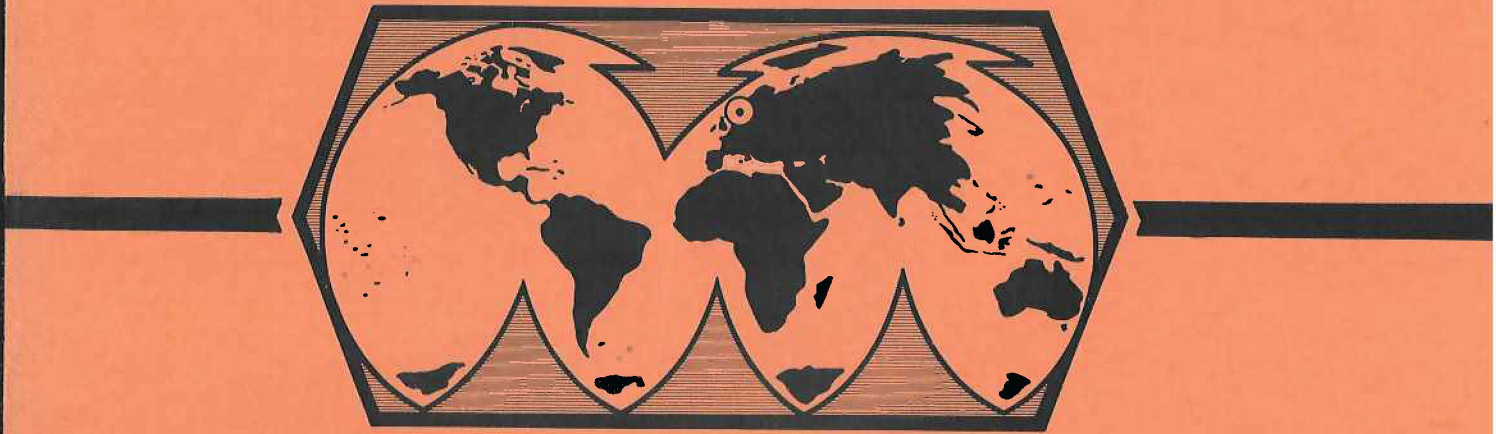

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B. A CHERNOV RANDOM MEDIUM MODEL

In the theory of body wave propagation in random media, the medium is characterized by its extent, and the mean square and autocorrelation of the refractive index. Based on a Gaussian type correlation function, Chernov (1960) has been able to establish a set of equations which makes it possible to estimate these parameters. The main assumptions are that the medium is statistically isotropic and homogeneous with small velocity fluctuations caused by large-scale inhomogeneities, the extent of the medium is large compared to the extent of the inhomogeneities and finally that the Rytov approximation is valid.

Important results of Chernov (1960) are the formulae for the transverse autocorrelation for phase and amplitude as a function of spatial lag. A drawback here is that the distance (ℓ) between instruments should not be much greater than the correlation distance (a). However, limitations of the above type of the Chernov theory have been removed by replacing approximately analytical integral solutions by numerical ones.

Another problem is that the observational data, i.e., relative travel time and logarithmic amplitude (logamplitude) of P-waves recorded across NORSAR, are likely to reflect not only forward wave scattering effects, but also deterministic effects like a locally dipping Moho. The latter effect is not accounted for in the Chernov (1960) theory, and unless removed may introduce a serious bias in the estimate of the random medium parameters.

Considering the crust and upper mantle beneath NORSAR as a Chernov medium, a least squares procedure has been utilized for estimating the relevant parameters. The starting point is to minimize the expression:

$$F = \mathrm{tr}(S_1-\Sigma_1)^2$$

where tr denotes trace and S_1 and Σ_1 are (N times N)-matrices, where N is number of instruments used. The S_1 and Σ_1 matrices are

$$S_1 = C\cdot S\cdot C' \qquad \text{and} \qquad \Sigma_1 = C\cdot\Sigma\cdot C'$$

where S is the observed covariance matrix for P-wave phase or logamplitude, while Σ is the theoretical covariance matrix. Σ is a function of the correlation distance $\underline{a}$, the mean square refractive index $\overline{\eta^2}$, the extent of the medium $\underline{L}$, and the measurement error θ. C is an N by N matrix and has the form

$$\begin{bmatrix} 1-\frac{1}{N} & -\frac{1}{N} & \cdots\cdots & -\frac{1}{N} \\ -\frac{1}{N} & 1-\frac{1}{N} & & -\frac{1}{N} \\ \cdots & \cdots & & \cdots \\ \cdots & \cdots & & \cdots \\ -\frac{1}{N} & -\frac{1}{N} & & 1-\frac{1}{N} \end{bmatrix}$$

C' is the transpose of C. The C matrix is introduced in order to remove deterministic wave propagation effects. For further details on this estimation procedure, see Christoffersson (1974).

For a subsection of NORSAR consisting of subarray 01A and 01B, the parameters (a,η,L,θ) which minimize F have been estimated (Berteussen et al, 1974), and it is found that the phase and logamplitude data give essentially the same results (see Table 1). The value obtained for $\underline{a}$ is approximately 15 km. The product $\overline{\eta^2}\cdot L$ occurs in almost all the expressions used; it is therefore difficult to separate the effect of these two parameters. That is, it is difficult to determine whether the medium is thick with small velocity variations or the medium is thin with large velocity variations. The results obtained so far indicate that L is at least 250 km and in this case η has a standard deviation of approximately 1.0%. For an array of the size considered here, aperture around 25 km, the Chernov theory is able to explain 74% of the variations in logamplitude and 94% of the variations in travel time anomalies. These values will certainly be diminished for a greater array where deterministic effects will have relatively more influence. It should also be stressed that the type of data used is amplitude and phase (measured at 0.7 Hz) of the very first cycle in the P-wave train.

As mentioned before, it is difficult to distinguish between a thick medium with small velocity variations and a thin medium with large variations in velocity. This point is also illustrated in the results obtained by different scientists. For example, for LASA Aki (1973) found L = 60 km and STD of η = 4% (Table 1). Capon (1974) obtained practically a two times larger value for L (136 km), but half as much velocity variations (STD of η = 2%). For NORSAR we again have doubled L ($L \geq 250$ km) but also reduced the velocity variations with a factor of two (STD of $\eta \approx 1\%$). These last values should, however, not be compared directly with those obtained at the LASA since the type of data and method of analysis are different. There may, of course, also be significant differences between the medium under LASA and that under NORSAR.

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TABLE 1

Calculated Chernov (1960) medium parameters for structures beneath the NORSAR and LASA arrays.

SOURCE	TYPE OF DATA			CHERNOV MEDIUM PARAMETERS		
	Array	Phase-Amp.	Freq. (Hz)	std. of refractive index (%)	Correlation Distance (km)	Extent of Medium (km)
This study	NORSAR	Phase	0.7	0.8	13.5	$\geq$ 250.0
This study	NORSAR	Amp	0.7	0.9	16.7	$\geq$ 250.0
Aki (1973)	LASA	Phase & Amp	0.6	4.0	10.0	60.0
Capon (1974)	LASA	Phase & Amp	0.8	2.0	12.0	120.0

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