

# NORSAR

ROYAL NORWEGIAN COUNCIL FOR SCIENTIFIC AND INDUSTRIAL RESEARCH

Scientific Report No. 4-73/74

## SEMIANNUAL TECHNICAL REPORT

### NORSAR PHASE 3

1 July–31 December 1973

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Kjeller, 11 January 1974



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F. SCATTERING OF P WAVES AT NORSAR

The standard linear deterministic model for travel time  $T$  (or logarithmic amplitude) at the  $i$ -th instrument of the array

$$T_i = T_0 + \vec{U} \cdot \vec{R}_i + \epsilon_i' \quad (F1)$$

where  $T_0$  is a constant,  $\vec{R}_i$  denotes position,  $\vec{U}$  denotes systematic (trend) effect and  $\epsilon_i'$  denotes random error. is considered unsatisfactory to explain the complex variation in these parameters. To obtain an improved description we have tentatively put up a linear stochastic model with random error component (see Fig. F1)

$$T_i = T_0 + \vec{U} \cdot \vec{R} + S_i + \epsilon_i \quad (F2)$$

The important term in this equation is the stochastic signal  $S_i$  which denotes the fluctuation in the parameter describing the wavefront according to eq. (F1).

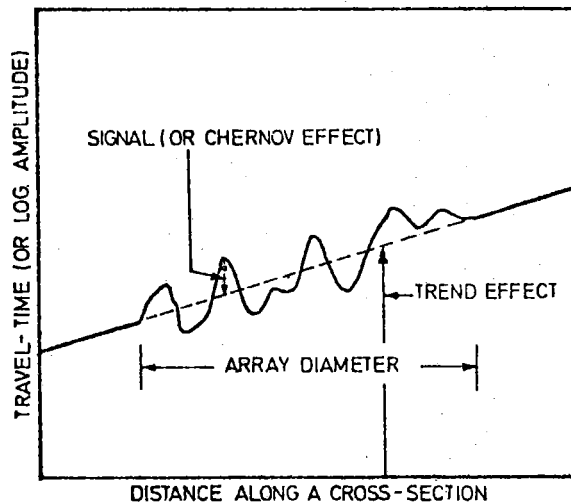


Fig. F1 Schematic decomposition of a stochastic model in a (systematic) trend effect and a correlated anomaly denoted signal.

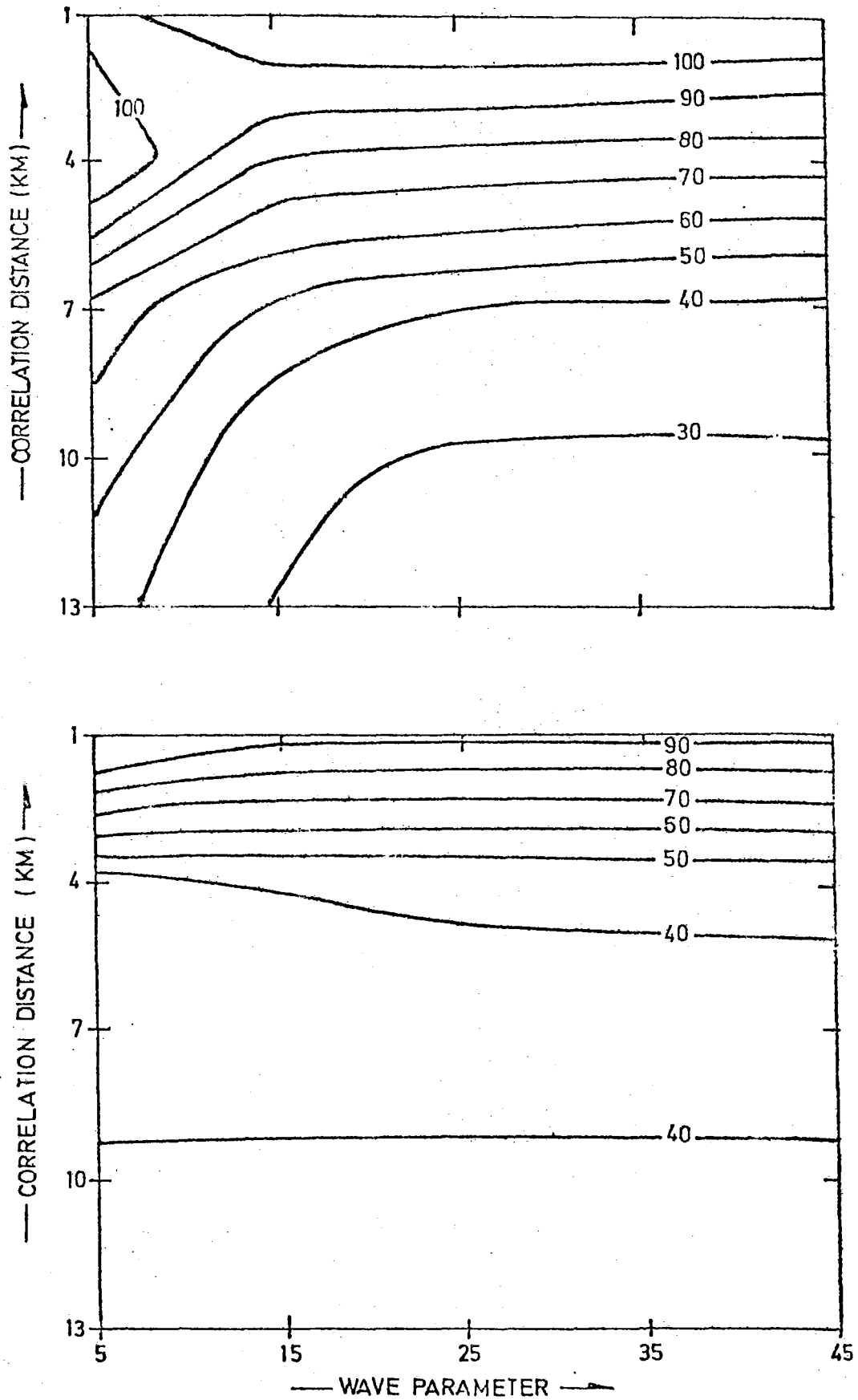


Fig. F2 Contour plot showing per cent variance of observed minus predicted travel times (lower) and logamplitudes (upper) using eq. (F2) relative to the same quantity using eq. (F1).

By stochastic we mean that  $E\{S_i \cdot S_i^*\} = C$  where the  $C$  is a covariance matrix with off-diagonal elements different from zero. Physically this means that the fluctuations in travel time and logamplitude are spatially correlated.

As an appropriate theoretical back-up we have chosen the scattering theory for acoustic waves in a random medium by Chernov (1960), where the matrix  $C$  is related to properties (wave parameter, correlation distance) of the inhomogenous medium through which the waves have been travelling. Adopting Chernov's formulae for large scale inhomogeneities and implementing the related covariance matrix into eq. (F2), we are in a position to use prediction for correlated observations to show that (see Fig. F2) the residual variances are reduced by more than 50% relative to eq. (F1).

It should be stressed that trying to evaluate the covariance matrix  $C$ , we are concerned with difficulties like estimating the "true" deterministic trend  $T_0 + \vec{U} \cdot \vec{R}$ , as well as separating the error residue  $\epsilon_i$  from the total anomaly  $S_i + \epsilon_i$

A consequence of the observations being correlated is that slowness estimates based on least squares estimation using eq. (F1) is less effective than using eq. (F2). In other words, when the array diameter is small and the spacing of sensors less than the correlation distance of the scattering medium, great care should be taken in interpreting velocity anomalies as structural discontinuities.

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REFERENCES

Chernov, L.A.: Wave propagation in a random medium, (Translated by R.A. Silverman), McGraw-Hill Book Company, New York, 1960.