# FINAL TECHNICAL REPORT <br> NORSAR PHASE 3 

1 July 1974-30 June 1975

Prepared by
K. A. Berteussen

Kjeller, 8. August 1975

Sponsored by<br>Advanced Research Projects Agency<br>ARPA Order No. 2551



APPROVED FOR PUBLIC RELEASE, DISTRIBUTION UNLIMITED

## Q. SOURCE-SPECTRAL SCALING AND CORNER FREQUENCIES

## Q. 1 Earthquake Love-wave Spectral Ratios

The scaling of earthquake source spectra has been known in an approximate way for some years, largely through the work of Haskell (1964, 1966) and Aki (1967). In the present work, the source factor of the far-field spectrum of Love waves is being studied using the $\omega$-square or $\omega$-cube model (Aki, 1967) as a starting point but not assuming the principle of similarity to the same extent. The spectral ratio method (Berckhemer, 1962) is applied wherein two or more earthquakes with virtually the same focal location are used and ratios of pairs of their Love-wave spectra are determined, cancelling out instrument and path factors.

Aki's (1967) source spectral model was of one parameter, presuming a one-to-one correspondence of $M_{s}$ and $m_{b}$. In this study a two-parameter model is assumed. This modified model allows for $M_{s}$ and $m_{b}$ to vary independently. It is expressed as

$$
\begin{equation*}
|A(\omega)|=\frac{k_{o} L^{\mathrm{P}}}{\left[1+k_{1} \omega^{2} L^{2}\right]^{\mathrm{S}}\left[I+\mathrm{k}_{2} \omega^{2} L^{2}\right]^{\frac{1}{2}}} \tag{0.1}
\end{equation*}
$$

where $p$ and $L$ are parameters and where $s=\frac{1}{2}$ or 1 according as the $\omega$-square or the $\omega$-cube model is chosen as a starting basis.

The expression for source spectral ratio then has four independent parameters. The estimate of these four parameters is then found from an algorithm which minimizes the variance of the model spectral-ratio values with respect to the values computed from the Fourier transforms of the two Love phases. This has been done assuming a modified $\omega$-square model as well as a modified $\omega$-cube model.

Although the theoretical spectral ratio is flat at high and low frequencies, its steepness at intermediate frequencies depends on the exponent of $\omega$ in the high-frequency asymptote. When we apply the model of equation (Q.l) (actually the ratio of two such expressions) to a group of deep-focus Bonin Islands earthquakes, we get a better fit for $s=1$ ( $\omega$-cube) than for $s=\frac{1}{2}$ ( $\omega$-square). An example of this is shown in Figures Q.1 and Q.2. In these figures the frequency range, which extended to 0.5 Hz for the Fourier transform output, is restricted to below 0.2 Hz because for higher frequencies one of the spectra is predominantly noise. The curves are extrapolated to 0.5 Hz but have no significance beyond 0.2 Hz . They should in fact level off just above the l-level.

From Equation (Q.1), the corner frequencies of an event, $\omega_{1}$ and $\omega_{2}$, are defined as $\left(k_{1} L^{2}\right)^{-\frac{1}{2}}$ and $\left(k_{2} L^{2}\right)^{-\frac{1}{2}}$, respectively. If these two corner frequencies (for one earthquake) differ, then we have (assuming $s=\frac{1}{2}$ ) an intermediate $\omega^{-1}$ behaviour as is present in models of Brune (1970) and Savage (1972), among others. For the present data, however, ${ }^{0} 1$ and ${ }^{0} 2$ come out virtually equal, implying that we have $\mathrm{k}_{1} \approx \mathrm{k}_{2}$ in equation (Q.1).

The corner frequencies determined so far agree in order of magnitude with those of the $\omega$-square and $\omega$-cube models (Aki, 1967) at magnitudes around $5 \frac{1}{2}$ to 6 , but there is some indication that $p$ in equation ( $Q .1$ ) may be greater than 3 , its value in Aki's models. This would imply a steeper corner-frequency locus, that is, less variation of corner frequency with earthquake size. This kind of behaviour has in fact been incorporated into Aki's (1972) revised model A.

## Q. 2 Explosion Source Spectra and Discrimination

Clearly, the fundamental difference between earthquakes and nuclear explosions is the difference in source mechanisms. This ought to manifest itself in the source spectra and indeed models of such for explosions (e.g., those of von Seggern and Blandford, 1972) differ significantly from those for earthquakes (e.g., those of Aki, 1967).

The Love-wave spectra from a group of Novaya Zemlya (presumed) explosions have been determined and an example of their spectral ratio is shown in Fig. Q.3. It is fundamentally different from the ratios of Figs. Q.l and Q.2. No attempt has been made to fit the modified $\omega$-square or $\omega$-cube models to this data -- these models assume, as a source, slip dislocation on a planar surface which is obviously inapplicable to an explosion source.

R.J. Brown

## REFERENCES

Aki, K. (1967): Scaling law of seismic spectrum, J. Geophys. Res., 72, 1217-1231.

Aki, K. (1972): Scaling law of earthquake source time-function, Geophys. J. R. Astr. Soc., 31, 3-25.

Berckhemer, H. (1962): Die Ausdehnung der Bruchfläche im Erdbebenherd und ihr Einfluss auf das seismische Wellenspektrum, Gerl. Beitr. z. Geophys., 71, 5-26.

Brune, J.N. (1970): Tectonic stress and the spectra of seismic shear waves from earthquakes, J. Geophys. Res.,75, 4997-5009.

Haskell, N. (1964): Total energy and energy spectral density of elastic wave radiation from propagating faults, Bull. Seism. Soc. Am., 54, 1811-1842.

Haskell, N. (1966): Total energy and energy spectral density of elastic wave rađiation from propagating faults, 2, A statistical source model, Bull. Seism. Soc. Am., 56, 125-140.

Savage, J.C. (1972): Relation of corner frequency to fault dimensions, J. Geophys. Res., 77, 3788-3795.
von Seggern, D., and R. Blandford (1972): Source time functions and spectra for underground nuclear explosions, Geophys. J.R. Astr. Soc., 31, 83-97.


Fig. Q.l Love-wave spectral ratio of two deep Bonin Islands earthquakes plotted versus frequency in Hz ; computed by Fourier analysis of long period NORSAR Love-wave beam and group-averaged with three points to a group (plus signs); the curve is the least squares best fit assuming the modified $\omega$-square model; S-wave corner frequency for the larger event (BI60) at 0.06 Hz .


Fig. Q. 2 Same as Fig. Q.1, but for the modified $\omega$-cube model; S-wave corner frequency of BI60 at 0.08 Hz .


Fig. Q.3 Same as Figs. Q.1 and Q. 2 for two Novaya Zemlya explosions; no model fit has been tried.

