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VII.2 Weighted Beamforming using Non-negative Weights

Considerable attention has been given to the weighted beamforming process of Christoffersson and Husebye (1974), where negative weights were allowed. Fyen et al (1975) showed that the presence of negative weights permitted additional statistical tests for use by the analyst in the signal-noise classification. The weights used in this beamforming process arose from the model

$$y_j = \gamma_j S + n_j \qquad j = 1,\dots,M \qquad (1)$$

where y_j is the recorded data at sensor j, S is the unknown signal and γ_j an unknown scaling factor which accounts for the observed amplitude variation at NORSAR. n_j is the residual noise of the model. The least squares estimators of this model are

$$\hat{S} = \sum \hat{\gamma}_j \, y_j \qquad (2)$$

where $\hat{\gamma}_j$ are the elements of the eigenvector corresponding to the largest eigenvalue of the matrix y'y, i.e., the variance-covariance matrix of the recordings (first principal component).

When $\hat{\gamma}_j$ takes negative values, it means physically that the sensor output has a phase shift of π relative to the beam. This in turn could occasionally lead to undesirable side effects as the phase shift option may result in projection of pure noise wavelets into the signal space. A project was therefore carried out where the estimated weights were permitted to take non-negative values only. The method of least squares leads to minimizing

$$F = \sum_{j=1}^{M} (y_j - \gamma_j S)'(y_j - \gamma_j S) - \lambda(\sum_{j=1}^{M} \gamma_j^2 - 1) \qquad (3)$$

where λ is the Lagrangian multiplier reflecting the restriction $\sum_{j=1}^{M} \gamma_j^2 = 1$. The minimum of F is found by an iteration procedure where only $\gamma \geq 0$ is permitted. This gives the same solution as the principal component method when all $\gamma_j > 0$ are positive. However, when some of the γ_j took negative values, the iteration procedure will give a solution which gives zero weights to sensors out of phase and slightly different values for others.

The procedure was applied to a set of mostly weak earthquakes with varying signal-to-noise ratios and extreme noise cases which trigger the on-line detector (false alarms). The results showed that non-negative weights do not give an opportunity to create additional statistical tests for signal-noise classification. However, when calculating signal-to-noise (SNR) ratios, the analysis showed that non-negative weights preserve the same SNR as the principal component procedure, and for extreme noise cases the SNR is reduced by at least 0.15 units on an average. In this region of SNR we know that the number of detections increases exponentially with decreasing SNR, so a reduction of SNR for extreme noise of 0.15 units is considered as a significant improvement relative to principal component beams, in terms of an SNR detector.

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References

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