NORSAR

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FINAL TECHNICAL SUMMARY

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VI. SUMMARY OF SPECIAL TECHNICAL REPORTS/PAPERS PREPARED

VI.1 Interference of Surface Waves

A seismic array such as NORSAR is well suited for the study of interference of surface waves which arrive at the array from more than one direction. However, there is a severe limitation in the fact that traditional array analysis methods such as the high-resolution frequency-wavenumber analysis technique (Capon, 1969) cannot resolve two wave trains when they arrive simultaneously from two directions (Bungum and Capon, 1974).

The alternative analysis technique presented here is based on studying the effect of the interference on the envelope of the resulting wave trains observed over the array. To this end, we start with a monochromatic wave with amplitude A, angular frequency ω and angular wavenumber vector $\bar{k}$, which at a station with position vector $\bar{r}$ can be represented by the expression

$$X(t) = A \exp[i(\omega t-\bar{k}\cdot\bar{r})] \tag{1}$$

If we add two such waves, with amplitudes A and $A(1-\varepsilon)$ and wavenumber vectors $\bar{k}_1$ and $\bar{k}_2$, respectively, it can be shown that the sum can be represented by the expression

$$X_s(t) = 2A[\cos\lambda - \frac{\varepsilon}{2}\exp(i\lambda)] \exp[i(\omega t-\bar{k}^*\cdot\bar{r})] \tag{2}$$

where

$$\lambda = \frac{\bar{k}_1-\bar{k}_2}{2}\cdot\bar{r} \quad , \qquad \bar{k}^* = \frac{\bar{k}_1+\bar{k}_2}{2}$$

For equal amplitude ($\varepsilon=0$) the expression is reduced to the more familiar one used for example by Bungum and Capon (1974). From equation (2) it can now be found that the (normalized) power loss in dB due to interference can be expressed as

$$\text{Loss(dB)} = -10 \log_{10} \left[\frac{(1-\varepsilon)\cos^2\lambda + (\frac{\varepsilon}{2})^2}{1-\varepsilon+(\frac{\varepsilon}{2})^2} \right] \quad (3)$$

If we now define x as the projection distance between seismometer and array center, measured along the resultant wave front, we find that

$$\lambda = |\bar{k}^*| \cdot x \cdot tg(\frac{\alpha}{2}) \quad (4)$$

where α is the angle between the two interfering wave trains. It now follows that the power loss can conveniently be plotted as a function of the quantity $|k^*| \cdot x$, given in radians.

For an initial testing of the interference pattern measured in this way at NORSAR, we have used a single seismometer recording of an earthquake from Japan. This recording has been used to simulate the simultaneous arrival of two wave trains with the same phase velocity (3.8 km/s), but with different azimuths (expressed through α) and amplitudes (expressed through ε). The measurements have been made at a period of 30 sec, and are based upon computation of the envelope for each of the 22 long period seismometer recordings at NORSAR. Fig. VI.1.1 here shows the effect of keeping a constant amplitude ratio of 1:1 while varying the azimuth difference from 30° to 120°, whereas Fig. VI.1.2 shows the results of keeping the azimuth difference constant ($\alpha=60^\circ$) while varying the amplitude ratio from 1:1 to 10:1. It is seen from these figures that there is a very good fit between measured and predicted power loss, and from the results we can draw the following conclusions, valid for 30 sec period surface waves recorded at an array with the size of NORSAR (diameter about 110 km):

1) The power loss due to interference is not serious for azimuth differences of 30° or less. At 60°, the loss may be as large as 20 dB.

2) The power loss due to interference is significant down to an amplitude ratio of about 5:1, although detectable at a ratio of 10:1.

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References

Bungum, H., and J. Capon (1974): Coda pattern and multipath propagation of Rayleigh waves at NORSAR, Phys. Earth Planet. Inter., 9, 111-127.

Capon, J. (1969): High-resolution frequency-wavenumber spectrum analysis, Proc. IEEE, 57, 1408-1418.

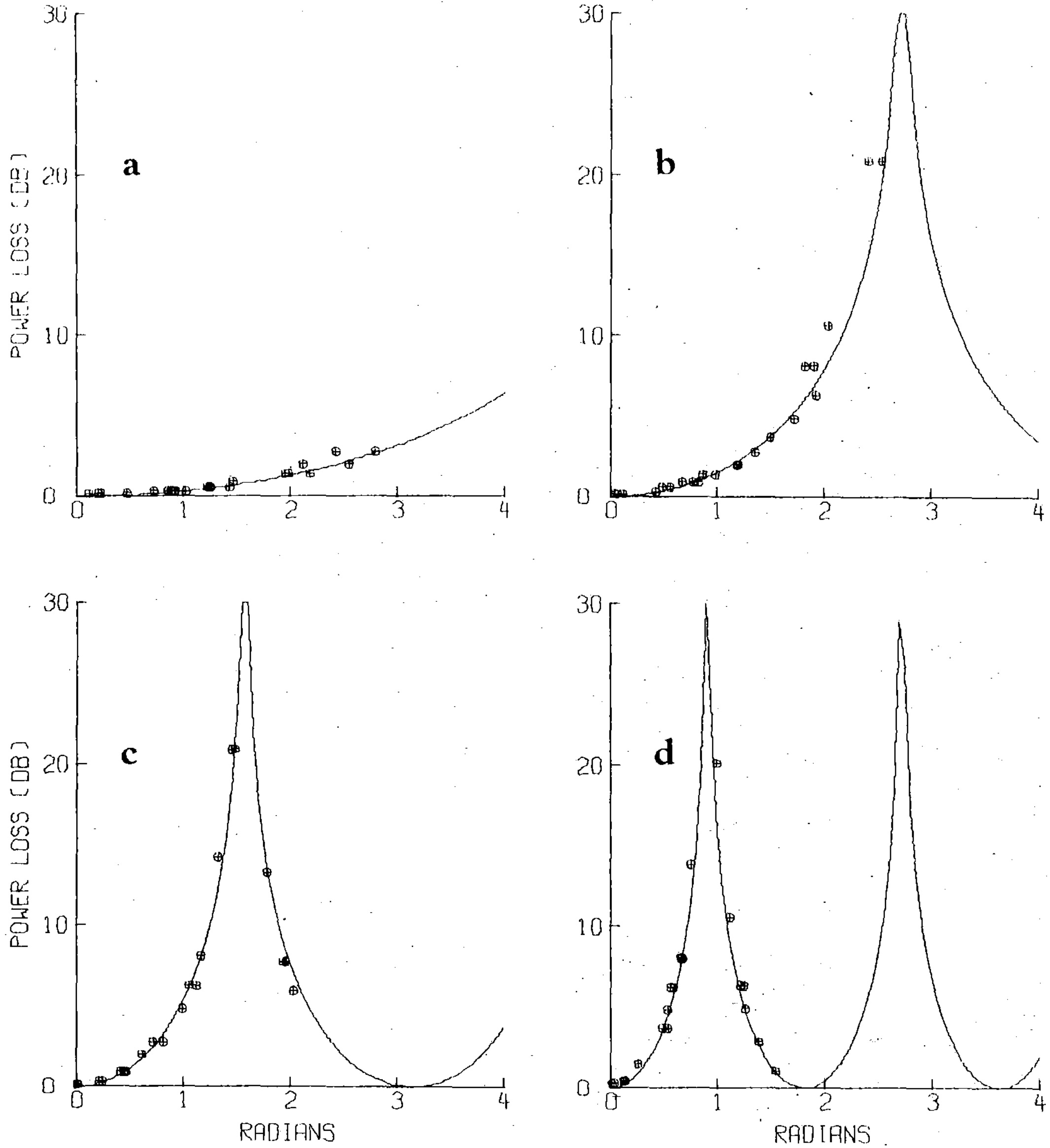

Fig. VI.1.1 Theoretical and observed power loss due to interference between two surface wave trains of equal amplitude arriving with an azimuth difference of a) 30°, b) 60°, c) 90° and d) 120°.

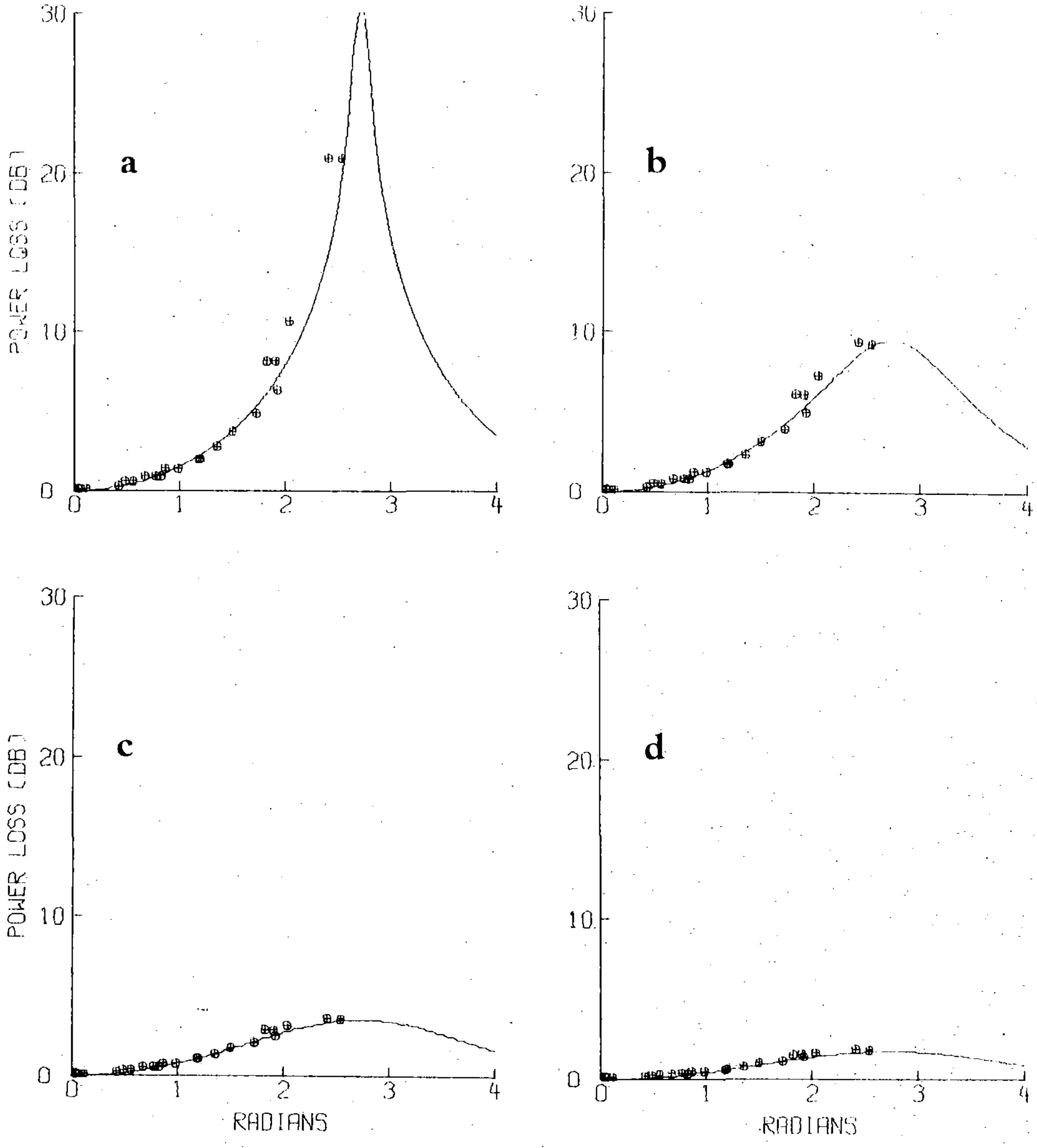

Fig. VI.1.2 Theoretical and observed power loss due to interference between two surface wave trains with an azimuth difference of 60°, and with amplitude ratios of a) 1:1, b) 2:1, c) 5:1 and d) 10:1.