

# NORSAR

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## FINAL TECHNICAL SUMMARY

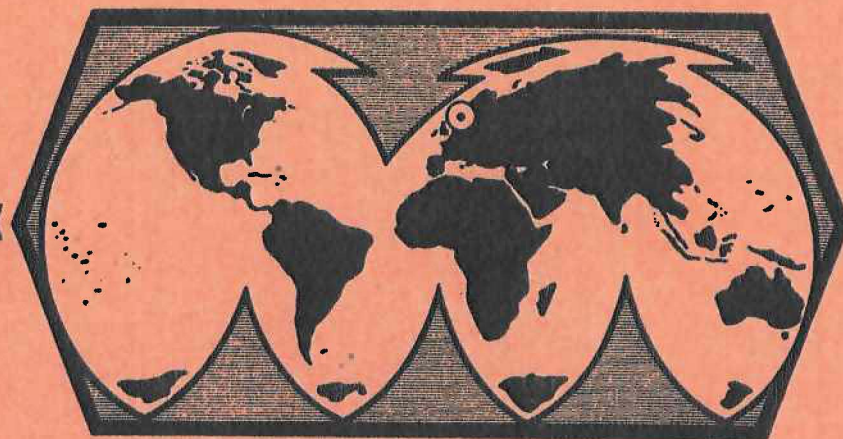
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## VI.9 A Criterion for Determining the Order of an AR Model

In geophysical time series modelling the observed time series are often sought approximated by autoregressive (AR) time series. This is for example the case in maximum entropy spectral estimation. The most used criterion for determining the order  $p$  of an autoregressive model

$$X(t) - a_1 X(t-1) - \dots - a_p X(t-p) = Z(t) \quad (1)$$

is the Akaike FPE criterion given as

$$\hat{FPE}(p) = \frac{N+p}{N-p} \hat{\sigma}_Z^2(p) \quad (2)$$

where  $\hat{\sigma}_Z^2 = \widehat{E|Z(t)|^2}$  is the estimated residual variance and  $N$  is the number of observations of  $X(t)$ . The FPE criterion picks the order  $p$  for which  $\hat{FPE}(p)$  takes its minimum. Based on a paper by Schwarz (1976) we suggest a new criterion, the AMBE (Asymptotic Maximum Bayes Estimator) which is defined as

$$\hat{AMBE}(p) = N \log \hat{\sigma}_Z^2(p) + p \log N \quad (3)$$

The integer  $p$  corresponding to the minimum value of  $\hat{AMBE}(p)$  is adopted as the order of the time series. The two criteria were compared in a number of simulation experiments. For the series given in Table VI.9.1 it is seen that the AMBE criterion performs considerably better than the FPE criterion. Further experiments are presently being conducted for higher order models. The preliminary results obtained suggest that the performance of the two criteria are approximately the same here.

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### References

- Sandvin, O.A. (1977): Modelling and Application of Parametric Time Series, Cand. real. thesis, University of Bergen, Norway.
- Schwarz, G. (1976): Estimating the dimension of a model. The Inst. of Advanced Studies, The Hebrew University of Jerusalem.

Table VI.9.1

20 realizations have been generated for each AR model. The mean value, standard deviation as well as the maximum and minimum value of the orders picked by the AMBE and FPE criteria, respectively, have been tabulated for each model.

Simulated AR-models							Mean Value of AMBE	Standard Deviation of AMBE	Mean Value of FPE	Standard Deviation of FPE	Max-Min Order from AMBE	Max-Min Order from FPE
Order	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$\sigma_z^2$						
2	1.60	-0.63	-	-	-	5.00	2.00	0.00	2.60	0.75	2 - 2	4 - 2
2	1.75	-0.72	-	-	-	10.00	2.25	0.42	2.80	0.67	3 - 2	4 - 2
2	0.30	0.60	-	-	-	5.00	2.15	0.39	2.90	1.35	3 - 2	7 - 2
3	0.80	0.82	-0.64	-	-	5.00	3.00	0.00	3.40	0.65	3 - 3	6 - 3
4	-0.30	1.05	0.18	-0.25	-	10.00	4.00	0.00	4.50	1.02	4 - 4	7 - 4
4	0.20	-1.16	-0.18	0.30	-	10.00	4.15	0.35	4.45	0.74	5 - 4	6 - 4
5	0.71	1.44	-0.80	-0.30	0.21	5.00	4.80	0.60	5.40	1.35	5 - 3	8 - 3
5	0.36	1.30	-0.54	-0.40	0.18	10.00	4.85	0.40	5.50	1.05	5 - 4	8 - 4