NORSAR ROYAL NORWEGIAN COUNCIL FOR SCIENTIFIC AND INDUSTRIAL RESEARCH

Scientific Report No. 2–76/77

SEMIANNUAL TECHNICAL SUMMARY

1 October 1976 – 31 March 1977

Edited by
H. Bungum

Kjeller, 15 March 1977

VII.10 A New Method of Spectral Estimation for Spatial Data

Geophysical data in general and seismological data in particular are often recorded both in space and in time. While a number of efficient statistical techniques exist for analysis of data observed in time, this is not so for spatial data. We propose here a new type of analysis for spatial variables observed on a grid. The proposed technique might be viewed as a generalization of the time-series autoregressive analysis, but there is also some justification for considering it a spatial extension of Burg's (1975) maximum entropy analysis.

For simplicity consider the case where we have a geophysical quantity $F(x_1,x_2)$ observed on a regular grid in the X_1-X_2 plane. We use the notation $S<x,y]$ for the vectors of integers $u=(u_1,u_2)$ which are such that $x_k \leq u_k \leq y_k$ for k=1,2 but $u \neq x$, and seek to model $F(x) = F(x_1,x_2)$ by a one-sided autoregressive model in the plane

$$F(x) - \sum_{y \in S<0,p]} a(y)\ F(x-y) = Z(x) \qquad (1)$$

where $Z(x) = Z(x_1,x_2)$ is a spatial white noise series. Corresponding to the time series case we obtain an estimate $\hat{f}(\lambda) = \hat{f}(\lambda_1,\lambda_2)$ of the two-dimensional power spectral density of $F(x)$ as

$$\hat{f}(\lambda) = \frac{\hat{\sigma}_z^2}{\left|1 - \sum_{y \in S<0,p]} \hat{a}(y)\ \exp\{-i[y,x]\}\right|^2} \qquad (2)$$

We have done some preliminary simulation experiments, using artificially generated data $F(x)$. In each case we computed $\hat{f}(\lambda)$ as well as an FFT power spectral estimate. Fig. VII.10.1 shows the results for a model of the form

$$F(x_1,x_2)=Z(x_1,x_2)+0.85\sigma_z \cos(1.5x_1+1.5x_2)+0.85\sigma_z \cos 1.5x_2 \qquad (3)$$

where we have two cosines embedded in two-dimensional spatial white noise. It is seen that the two spectral peaks are much sharper in the autoregressive case. This suggests that as in the time series case there may be cases where spatial autoregressive procedures are superior to FFT estimates. More tests are required to put this conclusion on a firmer basis.

D. Tjøstheim

REFERENCES

Tjøstheim, D. (1977): Autoregressive modelling and spectral estimation for spatial data. Some simulation experiments. Stanford Exploration Project, Department of Geophysics, Stanford University, Progress Report Vol. 11, to appear.

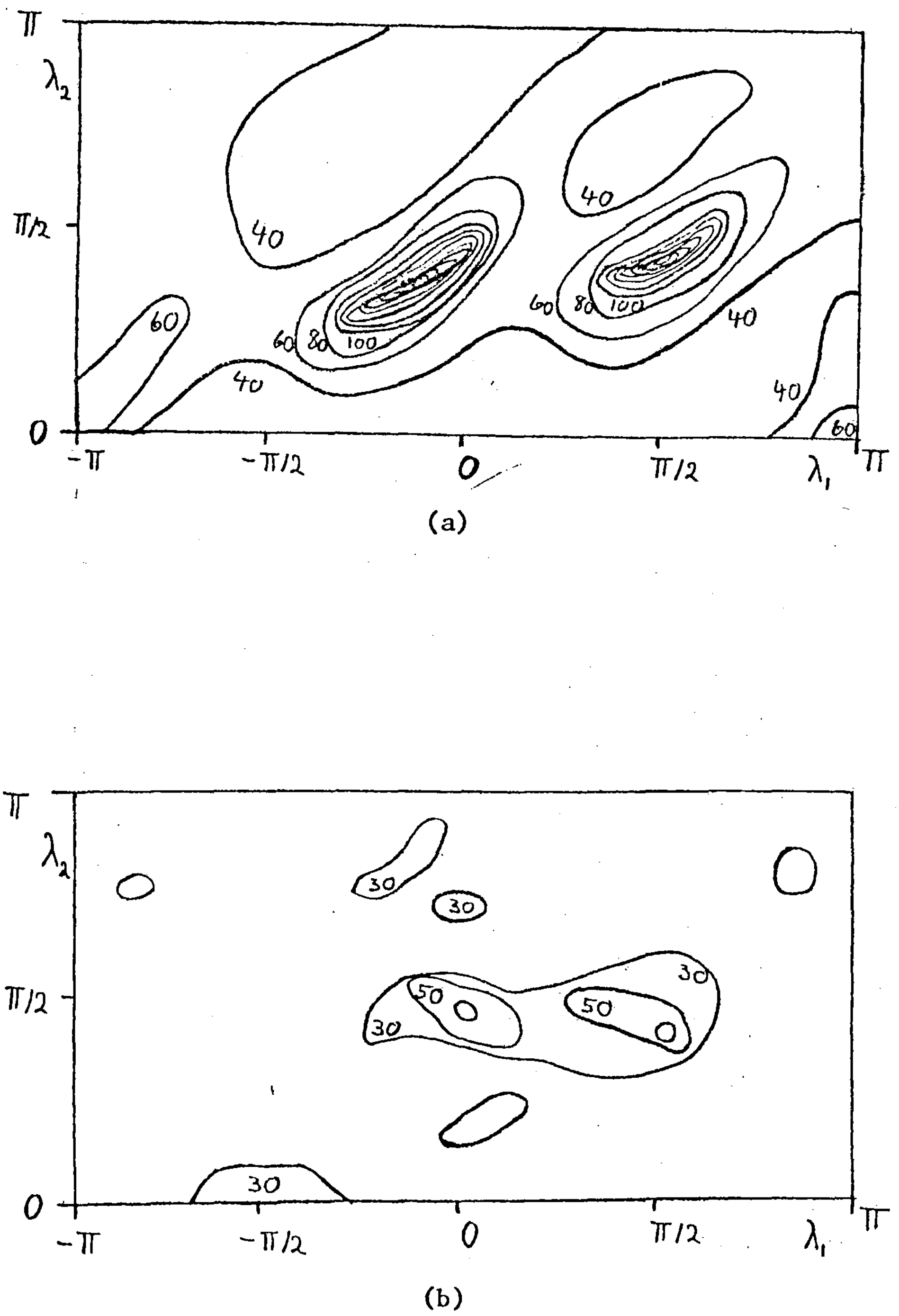

Fig. VII.10.1 Autoregressive AR(3,3) spectral approximation (a) and FFT approximation (b) of cosines embedded in spatial white noise.