NORSAR ROYAL NORWEGIAN COUNCIL FOR SCIENTIFIC AND INDUSTRIAL RESEARCH

Scientific Report No. 3-76/77

# FINAL TECHNICAL SUMMARY

1 April - 30 September 1977

Edited by

H. Gjøystdal

Kjeller, 31 October 1977

Sponsored by
Advanced Research Projects Agency
ARPA Order No. 2551

APPROVED FOR PUBLIC RELEASE, DISTRIBUTION UNLIMITED

## VI.10 A New Method of Spectral Estimation for Spatial Data. Part II

We have continued the experiments described in Tjøstheim (1977). In particular we have tried to implement a spatial version of the AMBE criterion given in Eq. (3) of Sandvin and Tjøstheim (1977). For a geophysical quantity $F(x) = F(x_1, x_2)$ defined on a regular grid in the $x_1$-$x_2$ plane the AMBE criterion is defined as

$$\hat{\mathrm{AMBE}}(p_1,p_2) = \left(\prod_{i=1}^{2} N_i\right) \log \hat{\sigma}_z^2(p_1,p_2) + \left(\prod_{i=1}^{2} (p_i+1)-1\right) \log \prod_{i=1}^{2} N_i \qquad (1)$$

where $p_i$ is the order and $N_i$ the number of observations in the $x_i$-direction i=1,2, and where $\sigma_z^2(p_1,p_2) = E|Z(x_1,x_2)|^2$ is the residual variance for an AR$(p_1,p_2)$ model. The pair of integers $(p_1,p_2)$ for which AMBE$(p_1,p_2)$ takes its minimum is adopted as the order of the AR model. As in Sandvin and Tjøstheim (1977) we did a number of experiments on artificially generated models. The results of Table VI.10.1 again suggest the superiority of the AMBE crition for 'ordinary' autoregressive models in the plane. The effect on spectral estimation is illustrated in Fig. VI.10.1 for model 3 of Table VI.10.1. It is seen that the overestimation of order brought about by the FPE criterion leads to a quite severe distortion of the spectrum. It should be noted also that the autoregressive AMBE spectrum (Fig.VI.10.1b) gives a much more accurate picture of the true spectrum than the FFT spectrum (Fig.VI.10.1d).

Unfortunately the AMBE criterion does not work very well on singular models of type

$$F(x_1,x_2) = Z(x_1,x_2) + A\cos(\alpha_1 x_1+\alpha_2 x_2) + B\cos(\beta_1 x_1+\beta_2 x_2) \qquad (2)$$

where we have two cosines embedded in two-dimensional spatial white noise. Especially for weak harmonic signals (small values of A and B) the low reduction in variance at each step of the autoregressive approximation implies that the last term in (1) dominates the first one for moderate values of $(p_1,p_2)$ and there is a tendency for the AMBE criterion to predict models on the 'edge', i.e., degenerate models of type $(0,p_2)$ or $(p_1,0)$. The spatial FPE criterion performs somewhat better in this respect.

D. Tjøstheim

References

Sandvin, O.A., and D. Tjøstheim (1977): A criterion for determining the order of an AR model. This report.

Tjøstheim, D. (1977): A new method of spectral estimation for spatial data. Semiannual Technical Summary, NORSAR Sci. Rep. No. 2-76/77, 64-66.

Table VI.10.1

Estimated orders $(p_1,p_2)$ as obtained from the minimum values of the spatial FPE and AMBE criteria, respectively. The correct orders are given in the uppermost line.

| Model | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Correct Order | (1,1) | (1,1) | (1,2) | (1,2) | (3,2) |
| FPE | (5,1) | (4,3) | (5,2) | (5,2) | (5,2) |
| AMBE | (1,1) | (1,2) | (1,2) | (5,2) | (3,2) |

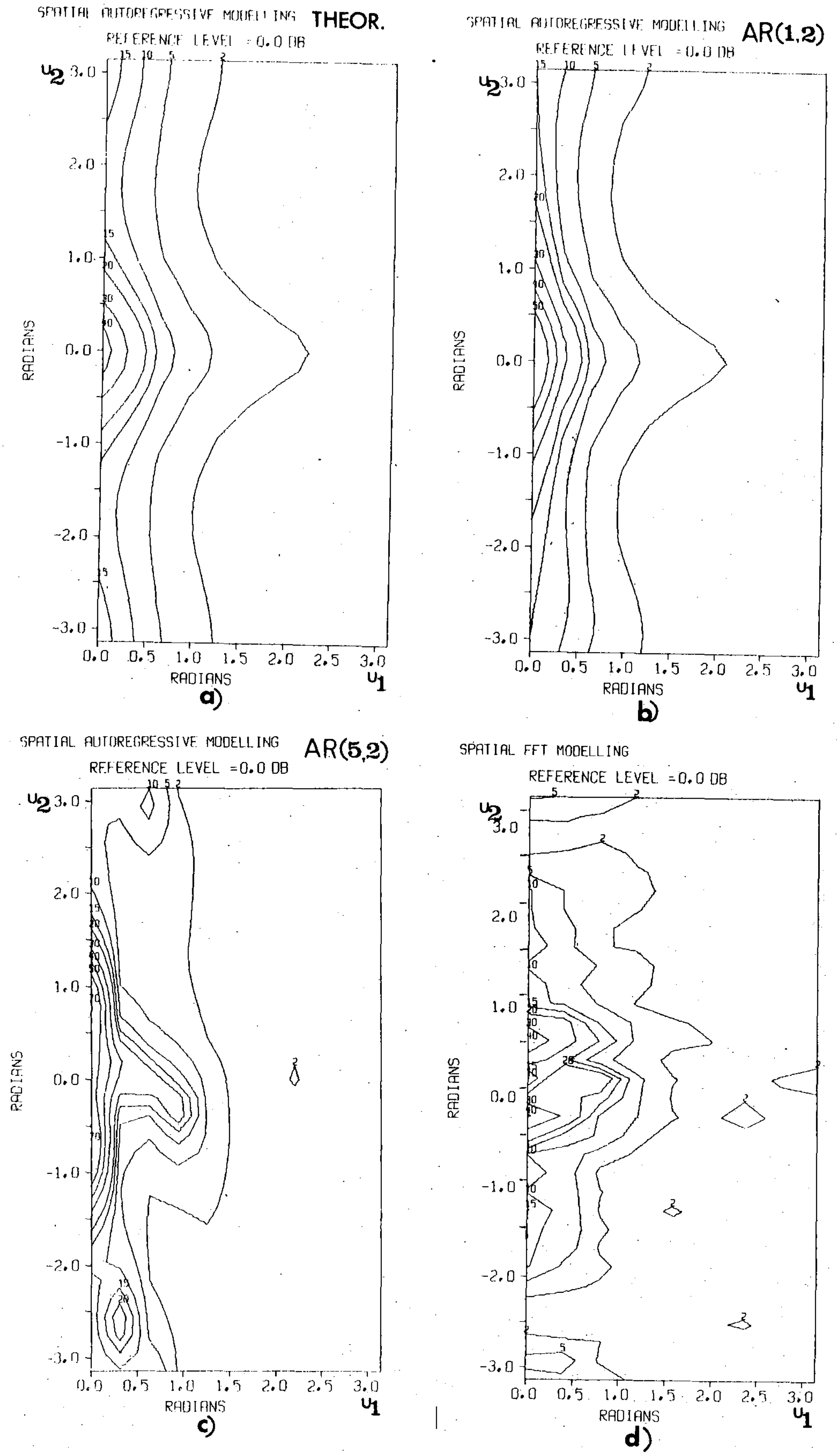


Fig. VI.10.1 a) Theoretical AR(1,2) spectrum, b) estimated AR(1,2) spectrum using the AMBE criterion, c) estimated AR(5,2) spectrum using the FPE criterion and d) estimated FFT spectrum for model 3 of Table VI.10.1 Model 3 is given as

$$F(x_1,x_2)-0.67F(x_1-1,x_2)-0.17F(x_1,x_2-1)+0.11F(x_1-1,x_2-1)-$$

$$0.17F(x_1,x_2-2)+0.11F(x_1-1,x_2-2) = Z(x_1,x_2).$$