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COMPUTATION OF SEISMIC RAY PATHS BETWEEN GIVEN SOURCE AND RECEIVER LINE IN A COMPLEX 3-D MODEL

by
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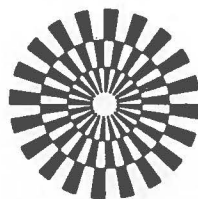


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1. Introduction

When applying ray-tracing procedures to 3-dimensional geological models, it is a well known fact that it is generally impossible to determine in advance where a reflected or refracted ray will happen to arrive at the model surface. In other words, there is no a priori way to determine which start direction should be used in order to find a ray connecting a given source-receiver pair. Several techniques have been proposed in order to solve the problem in an iterative way, f.ex. by consecutive tracing of rays according to some method which is expected to converge, or by starting with some 'arbitrary' path connecting the two positions, and then successively changing the path until the laws of reflection and refraction have been obeyed. (See Chander, 1975, 1977; Hubral, 1976; Julian and Gubbins, 1977; Shah, 1973; Sorrels et al, 1971.) Obviously, there will be a number of methods giving a satisfactory result; however, the basic problem lies in the fact that ray tracing especially for models with a certain degree of complexity - is relatively time consuming. Furthermore, the number of final ray paths connecting all combinations of sources and receivers of interest may in general be a rather large one in order to obtain a complete 'travel time picture' along the given profile. It is therefore of great importance that the number of ray-tracing operations needed to find these particular ray paths becomes as small as possible.

In this paper we will describe a method which may be used for 3-dimensional models of very high complexity. A problem when dealing with relatively complex models including numerous lateral discontinuities and great variations in interface curvatures is that it may be difficult to find procedures which are always converging towards the ray paths wanted. In addition, the situation may be complicated by so-called multipath effects which means that there may be different ray paths connecting the same source-receiver pair. Such effects give rise to discontinuities and/or reversals on the time-distance graphs, all of which should of course be adequately taken care of by the procedure applied.

Our method will take advantage of the fact that we generally have to do with a number of receivers rather than a single one, and that these receivers are distributed along some line on or near the surface. Thus, instead

of treating the receivers as single points, we shall consider a continuous 'receiver line' (which may or may not be curved) to which the rays shall be traced. By first obtaining travel time curves for this line, we may compute travel times for specific receiver points on the line simply by interpolation. The reason for considering a line rather than discrete receiver points is obviously that it is faster to let the ray approach a line than a point.

In the following we will give a rather general description of the method, leaving out details concerning model specifications, tracing of rays between the specified interfaces and so on. For such details we refer to Gjølstdal (1978b).

2. General Ray-tracing Procedure

2.1 Model Description

In order to trace seismic rays through a given model we have to adopt a mathematical description of the model, which may include such factors as:

- description of the interfaces where discontinuities occur in the seismic velocities
- description of points which shall be considered as diffractors
- description of the continuous velocity functions in the various blocks between the interfaces.

A very simple case which has been discussed in various papers is the isovelocity plane dipping layer model consisting of plane interfaces between constant velocity blocks. In such cases we need very few parameters for describing the model mathematically; in fact, we need 4 parameters for each layer: 3 for determining the interfaces and 1 for the interval velocity. When the model becomes more complex, including curved interfaces, lateral discontinuities in interface curvature, velocity variations in the medium between the interfaces, and so on, the number of parameters required increases drastically. For ex., the interfaces may be specified by a number of sample points making possible a functional fit in one or more regions of space. In addition, we have to adopt some

method for identifying the various velocity blocks which in general cannot be associated with one single interface as before.

In this paper we shall not give any detailed discussion of alternative model representations, but simply specify the model by what we may call a 'geological model vector' consisting of a list of all parameters required to obtain a complete mathematical description of the model:

$$\bar{G} = (g_1, \dots, g_{N_G}) \quad (1)$$

where N_G is the total number of model parameters.

A detailed method for description of complex 3-dimensional models may be found in a paper by GjØystdal (1978b).

2.2 Event (ray path) Description

In order to trace a ray through a predefined model, essentially two categories of parameters must be given:

- parameters specifying which type of ray path to follow when reflection/refraction occurs at the various model interfaces, together with the specification of an interface to which the ray shall be traced (receiver interface). This set of parameters will be called a 'ray type vector' or 'event vector' which can be written as

$$\bar{E} = (e_1, \dots, e_{N_E}) \quad (2)$$

N_E is the total number of parameters required to determine the type of ray path uniquely.

Fig. 1 gives some examples of various types of rays that may be considered.

- Initial parameters for the ray determining exactly which ray path to be traced. In most cases, the initial parameters are consisting of start coordinates \bar{x}_0 and a start direction unit vector \bar{m}_0 given relative to a 3D right handed rectangular coordinate system

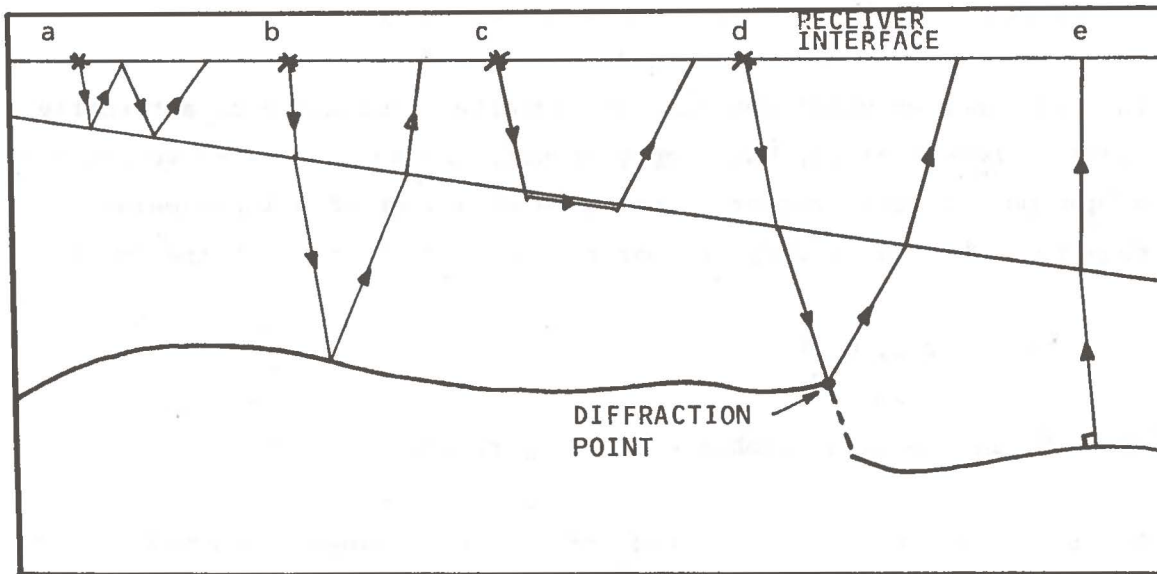


Fig. 1 Various types of rays which may be considered
a) Multiple ray path
b) Primary reflected ray path
c) Critically refracted ray path
d) Diffracted ray path
e) Normal incidence ray path

x_1, x_2, x_3 . However, as we shall see in a later section it may be advantageous to use slightly different initial parameters in some cases, and we will therefore restrict ourselves to define an 'initial parameter vector'

$$\bar{p}_0 = (p_{01}, p_{02}, \dots, p_{0N_p}) \quad (3)$$

In the case mentioned above we will therefore have

$$\bar{p}_0 = (x_{01}, x_{02}, x_{03}, m_{01}, m_{02}, m_{03}) \quad (4)$$

We may say that \bar{E} defines the ray family under consideration, while \bar{p}_0 selects a particular ray path from this family.

2.3 Ray-tracing Procedure - Formalistic Description

Knowing the model vector \bar{G} , event vector \bar{E} , and the initial values \bar{p}_0 , we may in principle trace the ray through the model until the receiver interface is reached (or the ray leaves the model). Each time the ray splits into two branches by reflection/refraction at a model interface, we know immediately from \bar{E} which branch to follow. The results of the complete ray-tracing process may be different for different purposes. Going on speaking in general terms, we may define a 'ray-tracing result vector'

$$\bar{q} = (q_1, \dots, q_{N_q}) \quad (5)$$

containing all the parameters from the ray-tracing process which we wish to extract, such as

- coordinates of the various ray/interface intersection points
- ray direction vectors in the various layers
- travel times and distances from initial point to various interfaces
- other parameters which may be associated with the ray path, f.ex., wavefront curvatures, etc.

We also define a 'tracing code index' I which is set equal to 0 if the ray-tracing process is successful and the receiver interface is reached. Otherwise, I may get a value $\neq 0$ indicating the reason for stopping the process before the final point was reached:

$$I = \begin{cases} 0 & \text{Successful process} \\ \neq 0 & \text{Unsuccessful process. Value of } I \text{ indicates the reason for failure.} \end{cases} \quad (6)$$

We may now adopt a formalistic description of the 'ray-tracing process' by the general expression

$$P[\bar{p}_0 | \bar{G}, \bar{E}] \rightarrow I, \bar{q} \quad (7)$$

where $P[]$ denotes the process, the components of \bar{p}_0 are the initial values of the process, and \bar{G} and \bar{E} the parameters of the process.

When \bar{G} and \bar{E} are given, and in addition $I = 0$, the elements of \bar{q} may be regarded as single-valued functions of the initial ray parameters:

$$q_i = q_i(\bar{p}_0) \quad i = 1, \dots, N_q \quad (8)$$

That is, the area of definition of the functions q_i is given by the set

$$S = \{\bar{p}_0 \mid I = 0\} \quad (9)$$

On the basis of the above formalistic description of the geological model and of the ray-tracing procedure, we may now be able to discuss certain aspects of the problem of searching for appropriate rays between predefined sources and receiver lines in the model. The advantage is that this can be done without going into any detail as to the model representations and the various kinds of ray-tracing procedures which can be thought of being applied.

3. Source/Receiver Configuration - Problem Definition

3.1 Source/Receiver Specification

The main objective of the method to be described is to simulate the shooting/recording process applied in seismic prospecting, and thereby compute important parameters that would be observed in the practical case. In order to do this for a given geological model, we therefore have to specify the source/receiver configuration we want to use, that is, we must give the coordinates of the source points and some mathematical description of the location of the receivers. For the moment we shall consider only one single source point, given by the coordinate vector \bar{x}_s . The receivers, however, will be assumed to be distributed along some continuous line or curve lying in some predefined 'receiver interface'. We shall not, until later, care about the position of each receiver point, but treat the whole 'receiver line' as a continuous recording feature being able to pick up waves in any location.

The receiver line shall be represented by a cubic spline function in a horizontal two-dimensional coordinate system x'_1, x'_2 which may be rotated relative to the horizontal axes x_1 and x_2 of the model system. It will be convenient to choose this 'receiver line system' in such a way that the receiver line essentially runs along the x'_1 -axis, f.ex., by letting the x'_1 -axis go through the initial and final point of the receiver line (see Fig. 2).

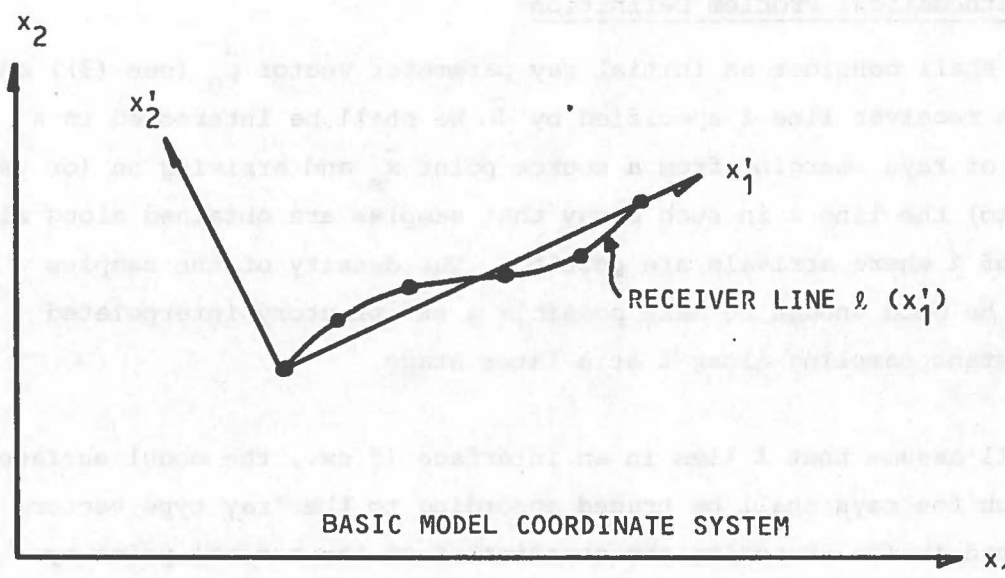


Fig. 2. Receiver line represented by a cubic spline function.

In mathematical terms, the receiver line may be expressed by the cubic spline function (see Ahlberg et al, 1967)

$$x'_2 = l(x'_1) \quad x'_{i1} \leq x'_1 \leq x'_{f1} \quad (10)$$

where x'_{i1} and x'_{f1} denote the initial and final x'_1 -coordinate of the receiver line.

To avoid going into more details as to the receiver line description, we simply give a 'receiver line vector'

$$\bar{L} = (\ell_1, \dots, \ell_{N_L}) \quad (11)$$

where N_L is the total number of parameters needed to describe the spline function completely, included the coefficients of transformation between the x_1x_2 - and the $x'_1x'_2$ -systems.

It should also be stressed that the receiver line need not itself lie in the $x'_1x'_2$ -plane, but rather in a predefined 'receiver interface' described by the model vector \bar{G} . Thus the spline function $\ell(x'_1)$ can simply be considered as the horizontal projection of the actual receiver line.

3.2 Mathematical Problem Definition

Now we shall consider an initial ray parameter vector \bar{p}_0 (see (3)) and a given receiver line ℓ specified by \bar{L} . We shall be interested in a family of rays emerging from a source point \bar{x}_s and arriving on (or very close to) the line ℓ in such a way that samples are obtained along all parts of ℓ where arrivals are possible. The density of the samples should be good enough to make possible a satisfactory interpolated equidistant sampling along ℓ at a later stage.

We shall assume that ℓ lies in an interface (f.ex., the model surface) to which the rays shall be traced according to the 'ray type vector' \bar{E} defined in (2). Denoting the coordinates of the arrival point by

$$\bar{x}_r = (x_{r1}, x_{r2}, x_{r3}) \quad (12)$$

we define the 'horizontal distance' from \bar{x}_r to ℓ by the expression

$$d = d(\bar{x}_r | \bar{L}) = x'_{r2} - \ell(x'_{r1}), \quad \bar{x}_r \in R \quad (13)$$

x'_{r1} and x'_{r2} are the coordinates of the arrival point \bar{x}_r relative to the 'receiver line system' $x'_1x'_2$ defined in Section 3.1. With 'distance' we simply mean distance along the x'_2 -axis. The distance d is expressed as a function of \bar{x}_r , provided that \bar{x}_r is located in an area R for which this distance exists. According to (10) we have

$$R = \{\bar{x}_r | x'_{i1} \leq x'_{r1} \leq x'_{f1}\} \quad (14)$$

Furthermore, since $\bar{x}_r \subset \bar{q}$ (see (5)), we have according to (8) that

$$\bar{x}_r = \bar{x}_r(\bar{p}_0) \quad (15)$$

and we may finally write

$$d = d(\bar{p}_0 | \bar{G}, \bar{E}, \bar{L}) \quad \text{if } \bar{x}_r(\bar{p}_0) \in R \quad (16)$$

We are now interested in finding initial ray parameter vectors \bar{p}_0 satisfying the requirement

$$|d(\bar{p}_0)| < \epsilon \quad (17)$$

in order to compute ray paths arriving closer to the receiver line than a given $\epsilon > 0$.

In practice, when searching for \bar{p}_0 's satisfying (17) we are always in the situation that we know some of the components of \bar{p}_0 and have to search for the rest. For example, we may consider a ray type like the second one shown in Fig. 1, i.e., a ray emerging from a given source point \bar{x}_s and is reflected in a given interface before returning to the surface. Obviously, the ray-path is uniquely determined by the initial ray parameter vector

$$\bar{p}_0 = (x_{s1}, x_{s2}, x_{s3}, m_{s1}, m_{s2}, m_{s3}) \quad (19)$$

where $\bar{m}_s = (m_{s1}, m_{s2}, m_{s3})$ is the ray direction unit vector in the source point. Since \bar{x}_s is given, and since only two of the components of \bar{m}_s are independent ($|\bar{m}_s| = 1$), the problem reduces to searching for only two parameters satisfying (17). As we shall see in a later section, also for other types of ray configurations (normal incidence rays, diffracted rays, critically refracted rays) we can reduce the problem to searching for just two initial ray parameters (p_1, p_2), although these two parameters will have different definitions in each case:

$$|d(p_1, p_2)| < \epsilon \quad (20)$$

Before turning to these special cases, we shall describe a general 'two-parameter search algorithm' which may thus be applied in the various types of problems to be discussed later.

4. A General Search Procedure for Sampling of Zero-value Contours of a Bivariate Scalar Function (ZCS-method)

In this section we will describe very shortly a general numerical search procedure which may be used in order to find initial ray parameters satisfying the requirement (20) in section 3.2. Here, the description will be restricted to giving the assumptions and input/output of the procedure, together with a short summary of the basic idea behind the method. For details, we refer to a separate paper by Gjølystdal (1978a).

Assume that a scalar function of two variables is defined on a region S of the x_1x_2 -plane

$$y = f(\bar{x}), \quad \bar{x} \in S \quad (21)$$

where $\bar{x} = (x_1, x_2)$ is a vector notation for the independent variables.

Assume further that S may be divided into regions S_i , $i=1, \dots, n$ in such a way that f is continuous on each S_i (that is, f is piecewise continuous on S).

The method to be described in this section was designed in order to calculate solutions of the equation

$$f(\bar{x}) = 0 \quad (22)$$

provided that such solutions exist somewhere in S . The procedure is based on the assumption that two operations are available:

Given an arbitrary point \bar{x}

- Determine if $\bar{x} \in S$
- If $\bar{x} \in S$, find $f(\bar{x})$.

Geometrically, the calculation of solutions of (21) corresponds to determination of intersection curves (or tangential curves) between the surface $y = f(\bar{x})$ and the \bar{x} -plane. In special cases one may think of having $f = 0$ in whole regions of the \bar{x} -plane, however, our method will be restricted to cases in which the solutions are made up by one or more continuous curves in S . Our aim will be to sample these curves (in the following denoted by C) using a given sampling interval ΔS , i.e., to calculate solutions of the form

$$\bar{x}_{ij}, \quad i=1, N_j, \quad j=1, M \quad (23)$$

Here \bar{x}_{ij} denotes sample point no. i of the j -th curve C_j , N_j is the number of samples for C_j and M the total number of separate curves.

The distance between each sample shall be approximately given by $|\bar{x}_{i+1,j} - \bar{x}_{ij}| \approx \Delta S$, except for the samples close to the end points of each C_j which in general shall have a smaller sampling interval in order to determine the discontinuities of the function more accurately. In case C_j is a closed curve, it should be sampled in such a way that the first and last sample points are identical, i.e., $\bar{x}_{ij} = \bar{x}_{N_j,j}$. See Fig. 3. It should also be noted that in the numerical procedure, the equation (21) must be replaced by the requirement

$$|f(\bar{x})| < \epsilon$$

i.e., a point \bar{x} is a satisfactory solution when giving functional values closer to zero than a certain predefined limit.

The method may be roughly described as follows. We start with a certain set of initial values (f.ex., on a given rectangular grid)

$$\bar{x}_i, \quad i=1, \dots, N_g \quad (24)$$

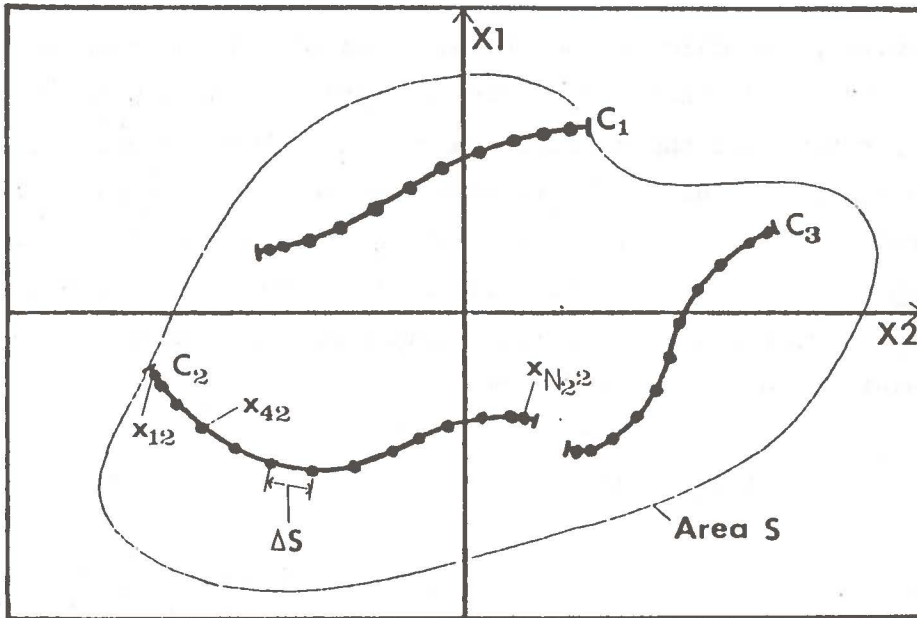


Fig. 3. The sampling of 'zero-value curves' C_j in the x_1x_2 -plane.

and select those \bar{x}_i satisfying

$$|f(\bar{x}_i)| < \epsilon_g \quad (25)$$

where ϵ_g is some preselected positive value. Thus we obtain a set of initial points \bar{x}_i giving functional values in a 'reasonable vicinity' of zero. Then, by using a special technique, we search along the direction of the gradient ∇f (in + or - direction depending on the sign of f) and thereby displace the points \bar{x}_i into new positions lying on (or sufficiently close to) the C-curves, so that

$$|f(\bar{x}_i)| < \epsilon \quad (26)$$

Finally, by starting in these displaced points, we search in directions normal to the gradient (i.e., along the tangent of the C-curves) and perform a continuous sampling of the C-curves. The sampling process is finished as soon as a discontinuity is reached or if the region S is

exceeded. As soon as a C-curve has been sampled, we go through the remaining start points \bar{x}_i and skip those which happen to belong to the C-curve just found. By choosing the initial grid dense enough we can, at least in principle, be sure of 'catching' all branches of the C-curves existing within the region S.

In the following sections the method will be referred to as the ZCS-method (Zero Contour Sampling method)

5. Application of the ZCS-method to the Problem of Finding Ray Paths Along a Preselected Receiver Line

In this section we shall demonstrate the application of the ZCS-method to different types of ray-tracing problems as mentioned in the end of section 3.2. In each case we will show that the number of initial ray parameters we have to search for may be reduced to 2 (see eq. (20)), making possible the use of the above method. However, before discussing the different types of ray paths separately in sections 5.2-5.5, we shall include some more general considerations which are valid in all these cases.

5.1 General Use of the ZCS-method

Assume for the moment that in a certain type of ray-tracing problem we have reduced the number of unknown initial ray parameters to 2. Consequently, we can split the initial ray parameter vector into two separate vectors:

$$\bar{p} = (p_1, p_2) \quad (27)$$

and

$$\bar{p}' = (p'_1, \dots, p'_{N_p-2}) \quad (28)$$

where the two-dimensional vector \bar{p} contains the 2 unknown components, and \bar{p}' contains the components of \bar{p}_0 which are predefined. The ray tracing process in (7) may now be written as

$$P[\bar{p}|\bar{p}', \bar{G}, \bar{E}] \rightarrow I, \bar{q} \quad (29)$$

and the elements of the result vector \bar{q} (see (8)) as

$$q_i = q_i(\bar{p}) \quad i=1, \dots, N_q, \quad I=0 \quad (30)$$

Also, the distance function d may be expressed as (see (14)-(16))

$$d = d(\bar{p}), \quad \bar{p} \in S \quad (31)$$

where

$$S = \{\bar{p} | \bar{x}_r(\bar{p}) \in R\} \quad (32)$$

Our problem is now to find points in the \bar{p} -plane satisfying the requirement

$$|d(\bar{p})| < \epsilon \quad (33)$$

These values of \bar{p} will of course correspond to rays arriving closer to the receiver line l than ϵ .

Using the ZCS-method described in Section 4 (see (23)), we may compute sampled curves C_j in the \bar{p} -plane consisting of points satisfying (33):

$$\bar{p}_{ij} \quad i=1, N_j, \quad j=1, M \quad (34)$$

Here \bar{p}_{ij} denotes sample point no. i of the j -th curve C_j , N_j is the number of samples for C_j and M the total number of separate curves.

By use of (34) we may now find some parametric representation of the C_j -curves:

$$\bar{p}_j(\rho) = (f_{1j}(\rho), f_{2j}(\rho)), \quad \rho \in [a_j, b_j] \quad (35)$$

where $f_{1j}(\rho)$ and $f_{2j}(\rho)$ are parametric functions for the components of \bar{p}_j , ρ is the chosen parameter and $[a_j, b_j]$ the interval of definition of ρ . The functions f_{1j} and f_{2j} may be easily found by fitting cubic spline functions to the sample points of (34). An example showing a possible parametric representation of a curve is given in Fig. 4, where the parameter ρ is chosen to be the angle between the p_1 -axis and the radius vector to the curve point.

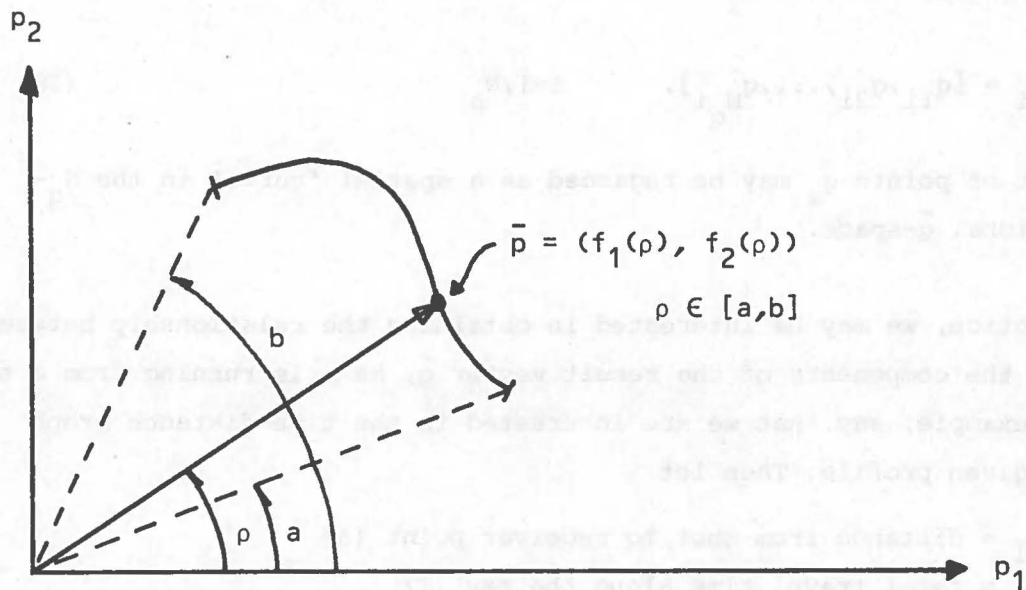


Fig. 4. Parametric representation of a curve.

In some cases it may also be appropriate to use one of the variables (f.ex., p_1) as the parameter and thus express the other as a function of the first by cubic spline fit.

Assume now that the functions f_{1j} and f_{2j} have been established for each curve C_j as shown in (35). Considering each curve C_j separately, we may omit the index j and write

$$\bar{p}(\rho) = (f_1(\rho), f_2(\rho)), \quad \rho \in [a, b] \quad (36)$$

When ρ is travelling from a to b , the radius vector $\bar{p}(\rho)$ will draw the continuous curve C between the points $\bar{p}(a)$ and $\bar{p}(b)$. By equidistant sampling of the interval $[a, b]$ we may obtain a set of samples

$$\rho_i, \quad i=1, N_\rho \quad (37)$$

where $\rho_1 = a$ and $\rho_{N_\rho} = b$. Since a given value of ρ , by (30), corresponds to one and only one value of the result vector \bar{q} , we thereby obtain a mapping of the interval $[a, b]$ into the vector space \bar{q} expressed by a set of components for each ρ_i :

$$\bar{q}_i = [q_{1i}, q_{2i}, \dots, q_{N_q i}], \quad i=1, N_\rho \quad (38)$$

The set of points \bar{q}_i may be regarded as a spatial 'curve' in the N_q -dimensional \bar{q} -space.

In practice, we may be interested in obtaining the relationship between two of the components of the result vector \bar{q} , as ρ is running from a to b . As an example, say that we are interested in the time-distance graph for a given profile. Then let

q_1 = distance from shot to receiver point (Δ)

q_2 = total travel time along the ray (T)

and compute these parameters for all ρ_i , $i = 1, \dots, N_\rho$. If all result vectors (q_{1i}, q_{2i}) are plotted into the $q_1 q_2$ -plane ($T\Delta$ -plane), we obtain a sampled curve in this plane, which is the wanted time-distance graph. This graph may contain discontinuities, break points, or reversals, which means that a division into several separate branches may be necessary if a functional fit is wanted in the $q_1 q_2$ -plane.

As a general conclusion we may say that by use of the parameter set ρ_i , we may find a functional relationship between any pair (q_j, q_k) , $j, k=1, \dots, N_q$, $j \neq k$, or between any pair (p_j, q_k) $j=1, 2$, $k=1, \dots, N_q$, by choosing one of the two parameters as the independent variable, and fitting functions (f.ex., cubic splines) to the data points obtained. By use of these functions we may easily interpolate to find the value(s) of the dependent variable for any given value of the independent variable. This corresponds to finding values of T for any value of Δ in the example above.

Fig. 5 shows a schematical illustration of the transformation process from the interval $[a,b]$ to the q_1q_2 -plane.

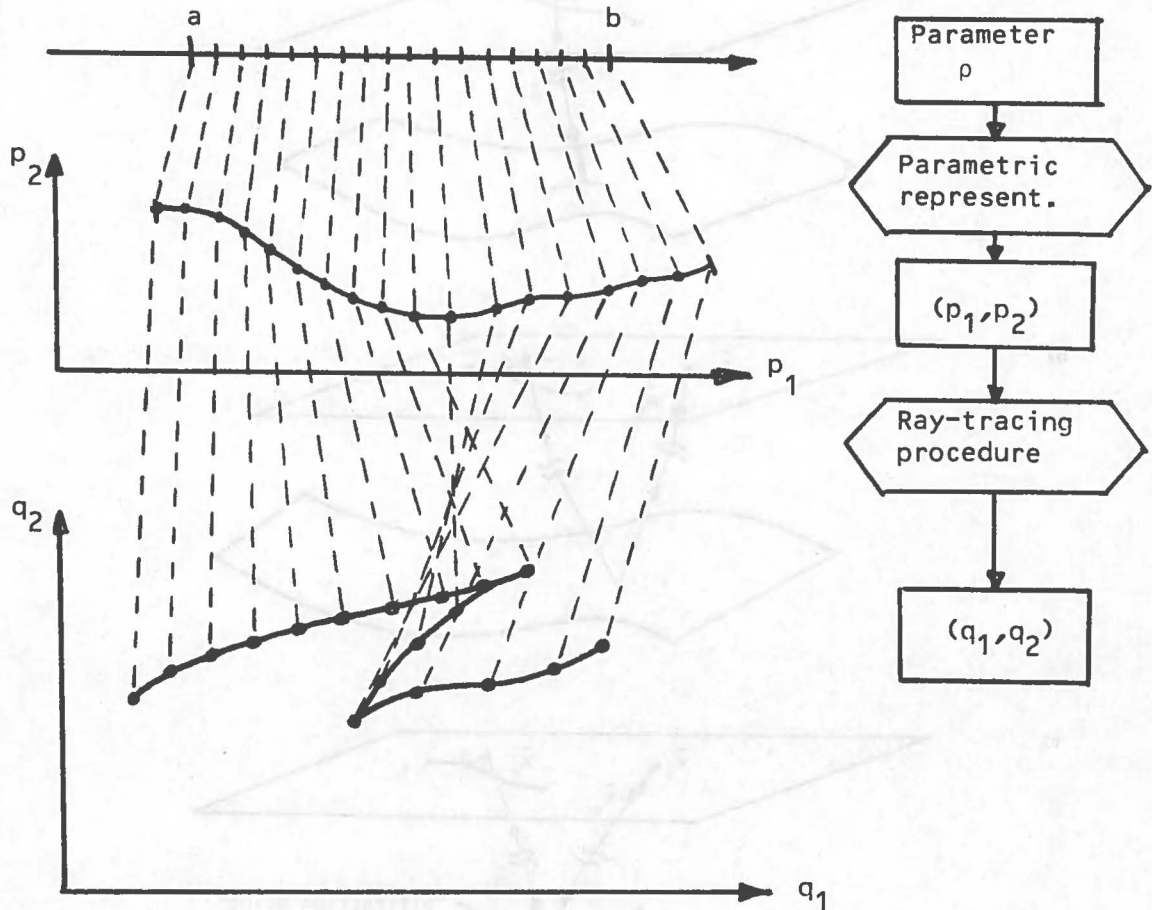


Fig. 5 Illustration of the transformation process from the parameter interval $[a,b]$ to the q_1q_2 -plane.

5.2 Tracing of Normal Incidence Ray Paths

To simulate zero-offset observations (source and receiver in the same point) we are interested in computing the rays emerging from a source point and returning to the same point after having been reflected normally from a given interface (see Fig. 6a). The simplest way to do this is to start in the reflection interface in the direction of the interface normal and trace the ray to the receiver interface containing the line ℓ . The problem here is to choose the coordinates of the

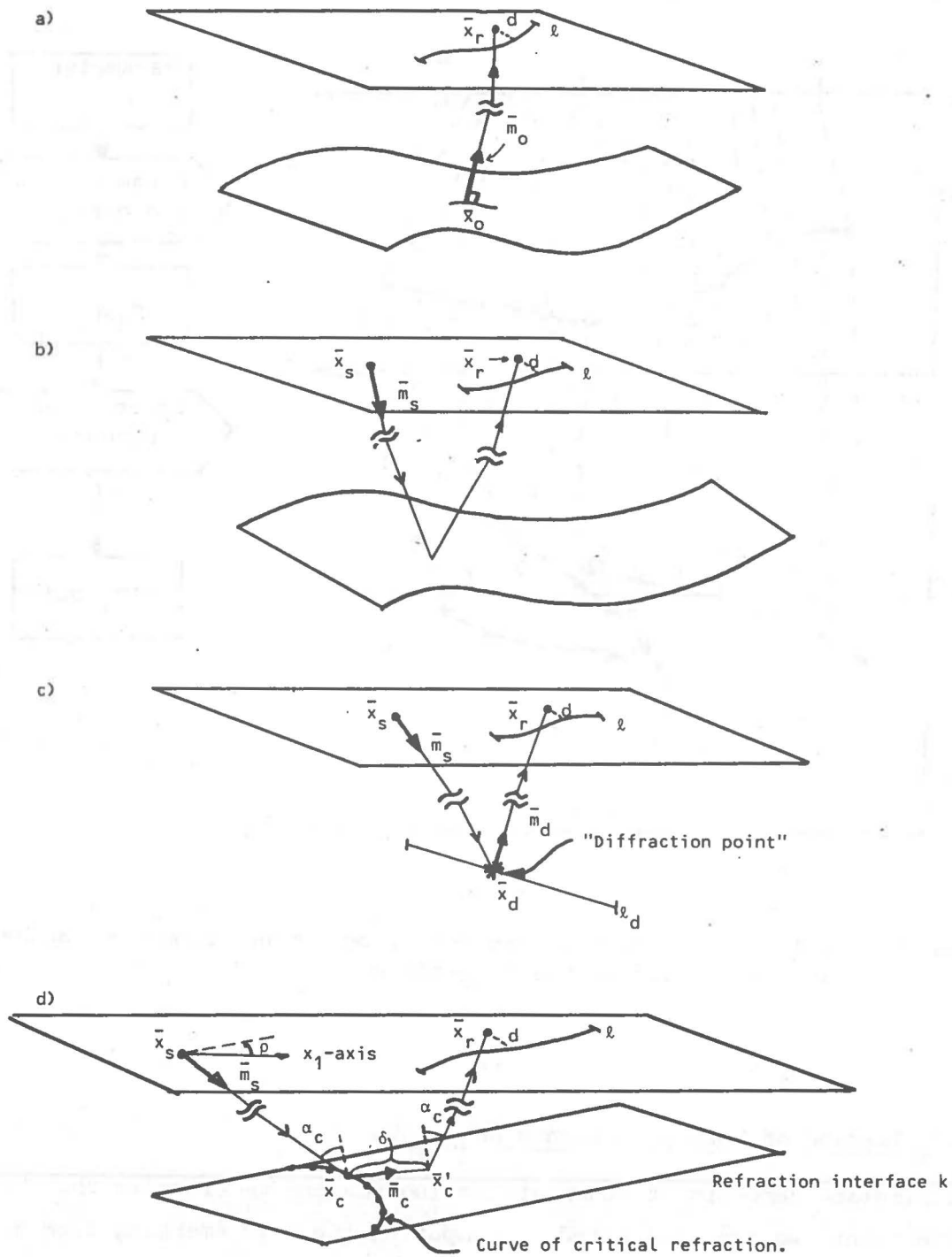


Fig. 6 Illustrations of the 4 different cases to be discussed:

- a) Normal incidence ray
- b) Non-zero offset ray
- c) Diffracted ray
- d) Critically refracted ray.

start point \bar{x}_0 in such a way that the ray arrives close enough to ℓ .

Choosing the two components (x_{01}, x_{02}) as the two unknown variables of (27), we may easily show that the remaining parameters may be computed from these two. In fact, since \bar{x}_0 is located in a given interface, we have that

$$x_{03} = h(x_{01}, x_{02}) \quad (39)$$

where h is a functional representation of the interface (given in \bar{G} , eq. (1)). Furthermore, since the initial direction vector \bar{m}_0 is identical to the interface normal, we can write

$$\bar{m}_0 = \bar{m}_0(x_{01}, x_{02}) \quad (40)$$

and the independent and given initial ray parameter vectors of (27) and (28) are respectively

$$\bar{p} = (x_{01}, x_{02}) \quad (41)$$

$$\bar{p}' = (\text{no components}) \quad (42)$$

The vector \bar{p}' contains no components, since in this case $\bar{p} = (x_{01}, x_{02})$ defines the complete ray path uniquely.

The distance function d is thus on the form of eq. (31) and the ZCS-method may be applied to search for the horizontal coordinates (x_{01}, x_{02}) in the given interface from which NIP-rays arrive along the receiver line ℓ .

The procedure applied to find the value of the function $d = d(x_{01}, x_{02})$ is straightforward:

- Trace a ray with start values \bar{x}_0 and \bar{m}_0 until receiver interface is reached to find \bar{x}_r (see (29)).
- Compute d according to (13).

5.3 Tracing of Non-zero Offset Reflected Ray Paths

In this case the source point \bar{x}_s is given, and rays shall be traced from \bar{x}_s to a given reflecting interface and back to the receiver interface containing ℓ (see Fig. 6b). The problem now is to search for initial directions \bar{m}_s giving arrival points sufficiently close to ℓ . In analogy with (41) and (42) we set

$$\bar{p} = (m_{s1}, m_{s2}) \quad (43)$$

$$\bar{p}' = (\bar{x}_s) \quad (44)$$

Here $\bar{x}_s = (x_{s1}, x_{s2}, x_{s3})$ is given and

$$m_{s3} = \pm \sqrt{1 - (m_{s1}^2 + m_{s2}^2)} \quad (45)$$

since $|\bar{m}_s| = 1$.

Again the ZCS-method may be used to find the initial direction vectors giving arrivals satisfying the usual requirement $|d(m_{s1}, m_{s2})| < \epsilon$.

The computation of the function $d = d(m_{s1}, m_{s2})$ is again straightforward:

- Trace a ray with start values \bar{x}_s and \bar{m}_s until reflecting interface is reached and back to receiver interface to find \bar{x}_r .
- Compute d according to (13).

5.4 Tracing of Diffracted Ray Paths

In this case we shall assume that a diffraction point \bar{x}_d is given, f.ex., at a prominent edge of a fault system. In order to apply the search method, we simply define an arbitrary line segment containing \bar{x}_d , denoted by ℓ_d , which is supposed to be described in the model vector \bar{G} given in (1).

Here we shall consider the following problem: Given a source point \bar{x}_s and a diffraction point \bar{x}_d belonging to the line ℓ_d . Find rays starting in \bar{x}_s , being diffracted in \bar{x}_d and arriving along the receiver line ℓ as usual (see Fig. 6c). The situation is very similar to the one we discussed in section 5.3, except for the ray segment from \bar{x}_s to \bar{x}_d which

must be common for all rays diffracted in \bar{x}_d . To find this ray path, we simply treat the diffractor line ℓ_d as a receiver line and search for initial directions \bar{m}_s for rays starting in \bar{x}_s and arriving sufficiently close to ℓ_d . This problem is identical to the one discussed in section 5.3. Then, using the interpolation method described in section 5.1, we may find the ray path (with associated parameters) arriving in the particular point \bar{x}_d of ℓ_d .

Having computed the ray path from \bar{x}_s to \bar{x}_d we may now treat \bar{x}_d as the source point and search for directions \bar{m}_d of rays arriving along the receiver line ℓ . We thus have

$$\bar{p} = (m_{d1}, m_{d2}) \quad (46)$$

$$\bar{p}' = (\bar{x}_s, \bar{x}_d) \quad (47)$$

Here \bar{x}_s and \bar{x}_d are given and

$$m_{d3} = \pm \sqrt{1 - (m_{d1}^2 + m_{d2}^2)} \quad (48)$$

The computation of the function $d = d(m_{d1}, m_{d2})$ will now be as follows:

- Trace a ray with start values \bar{x}_d and \bar{m}_d from diffraction point to receiver interface to find \bar{x}_r .
- Compute d according to (13).
- To obtain the complete ray path from \bar{x}_s to ℓ , add the ray portion $\bar{x}_s - \bar{x}_d$ to the one just found.

5.5 Tracing of Critically Refracted Ray Paths

In this case we shall be interested in rays emerging from a given source point \bar{x}_s in a direction \bar{m}_s which is such that the ray will be critically refracted along a given plane interface k . Furthermore, the rays shall leave this plane interface under the critical refraction angle and arrive sufficiently close to the receiver line ℓ as before (see Fig. 6d).

The first problem is to find two independent parameters which uniquely determine the entire ray path, so that we can make the usual search for

arrivals along ℓ . Obviously, we cannot choose two independent components of \bar{m}_s as before since we have the requirement that the ray shall intersect the interface k under the critical angle of refraction. In fact, we have only one degree of freedom in choosing components of \bar{m}_s . F.ex., for a given component m_{s1} , the assumption of critical refraction requires that the other two components are functions of m_{s1} . Our first task will be to find this functional relationship.

Again we will use the ZCS-method to search for values of \bar{m}_s giving critical refractions along k . Denoting the angle of incidence at k of an arbitrary ray by α , and the critical angle of refraction by α_c , we may express the difference between the sine of the angles as

$$d_\alpha = \sin \alpha - \sin \alpha_c = \sqrt{m_{k1}^2 + m_{k2}^2} - \frac{v_1}{v_2} \quad (49)$$

Here \bar{m}_k is the direction unit vector of the incident ray at interface k , and v_1 and v_2 the wave velocity immediately above and below k in the refraction point \bar{x}_k . Since the right side of (49) depends only on the ray path, and since $|\bar{m}_s| = 1$, d_α may be considered as a function of the two variables (m_{s1}, m_{s2})

$$d_\alpha = d_\alpha(m_{s1}, m_{s2}) \quad (50)$$

Using the ZCS-method, we may now search for points (m_{s1}, m_{s2}) satisfying the requirement

$$|d_\alpha| < \epsilon_c$$

where ϵ_c is some small positive value. We may thus obtain sampled curves C in the m_{s1}, m_{s2} -plane as shown in (34) and consequently the components m_{s1} and m_{s2} may be given in a parametric representation by fitting a cubic spline function to the sample points:

$$m_{s1} = m_{s1}(\rho) \quad (51)$$

$$m_{s2} = m_{s2}(\rho) \quad \rho \in [a, b] \quad (52)$$

$$m_{s3} = \pm \sqrt{1 - (m_{s1}^2 + m_{s2}^2)} \quad (53)$$

In this case it may be appropriate to choose ρ as the angle between the x_1 -axis and the horizontal projection of the ray emerging from \bar{x}_s , as shown in Fig. 6.

Denoting the direction unit vector of the critically refracted ray (parallel to k) by \bar{m}_c and the coordinates of the critical refraction point by \bar{x}_c , we immediately have

$$\bar{x}_c = \bar{x}_c(\rho) \quad (54)$$

$$\bar{m}_c = \bar{m}_c(\rho) \quad \rho \in [a, b] \quad (55)$$

Now, ρ is shown to determine the ray path uniquely from \bar{x}_s to \bar{x}_c . To determine the ray path uniquely from \bar{x}_c to the arrival point \bar{x}_r we may define a new variable δ as the distance from \bar{x}_c along \bar{m}_c to the second critical refraction point \bar{x}'_c where the ray is determined to leave the interface k (see Fig. 7).

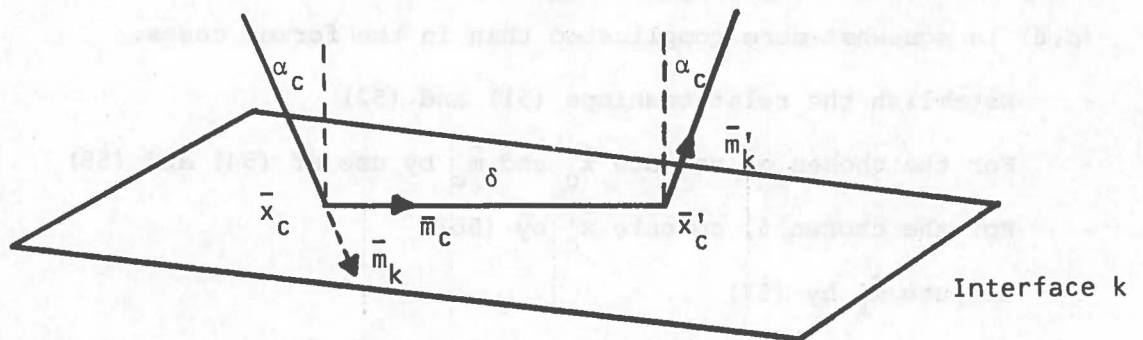


Fig. 7. Geometry of critically refracted rays.

We then have

$$\bar{x}'_C = \bar{x}_C + \delta \cdot \bar{m}_C \quad (56)$$

The direction of the critically refracted ray in \bar{x}'_C is denoted by \bar{m}'_k which may be computed from the vector cross product equation

$$\bar{m}_C \times \bar{m}'_k = \bar{m}_k \times \bar{m}_C \quad (57)$$

This gives 3 component equations determining \bar{m}'_k .

We have now obtained two independent parameters (ρ, δ) which define the critically refracted ray-path uniquely. The initial ray parameters of (27) and (28) now become

$$\bar{p} = (\rho, \delta) \quad (58)$$

$$\bar{p}' = \bar{x}_s \quad (59)$$

Here \bar{x}_s is given and \bar{m}_s may be computed from ρ according to (51)-(53).

The problem now reduces to the usual problem of searching for pairs (ρ, δ) satisfying

$$|d| = |d(\rho, \delta)| < \epsilon \quad (60)$$

The procedure of computing the functional value of d , given a pair (ρ, δ) is somewhat more complicated than in the former cases:

- Establish the relationships (51) and (52)
- For the chosen ρ , compute \bar{x}_C and \bar{m}_C by use of (54) and (55)
- For the chosen δ , compute \bar{x}'_C by (56)
- Compute \bar{m}'_k by (57)
- Trace a ray from \bar{x}'_C with initial direction \bar{m}'_k until the receiver interface is reached to find \bar{x}_r
- Compute d according to (13).

6. Some ray-tracing examples

6.1 Description of model

In this section we shall show some examples of results obtained by applying the methods in practice. For convenience, we choose a relatively simple 3-dimensional model as shown in Fig. 8.

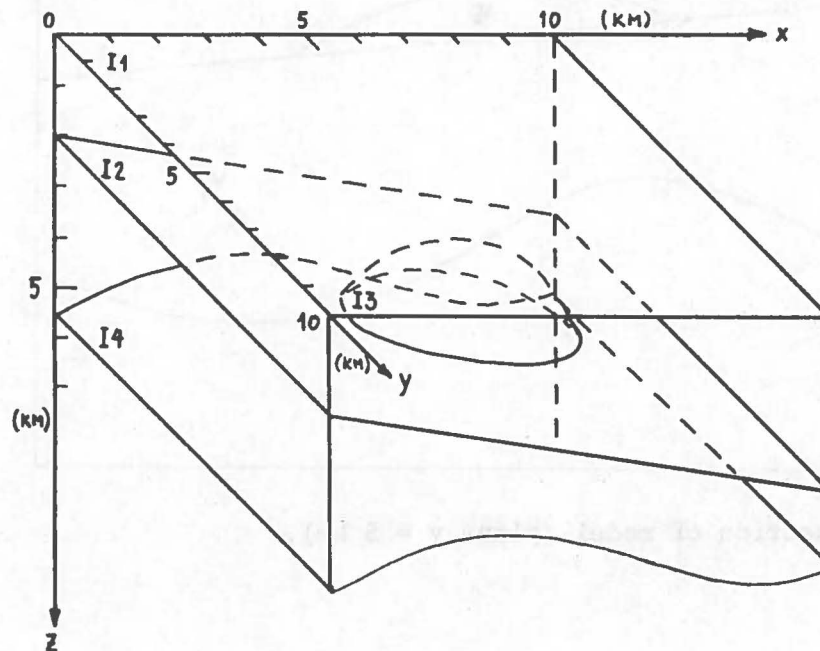


Fig. 8. Model example.

The model consists of 4 interfaces, denoted by I_1 - I_4 , and 3 velocity blocks, V_1 - V_3 , described as follows:

- I_1 - Plane horizontal interface (xy-plane)
- I_2 - Plane interface dipping in the x-direction
- I_3 - 3D curved interface bounded below by interface I_2
- I_4 - Cylindrical interface in the x-direction (constant in the y-direction).

- V_1 - $v = 2$ km/s
- V_2 - $v = 3$ km/s
- V_3 - $v = (2 + 0.5 \cdot z)$ km/s

Fig. 9 shows a cross section of the model in the plane $y=5$ km. For a more detailed description of this particular model, we refer to Gjølstdal, 1978b.

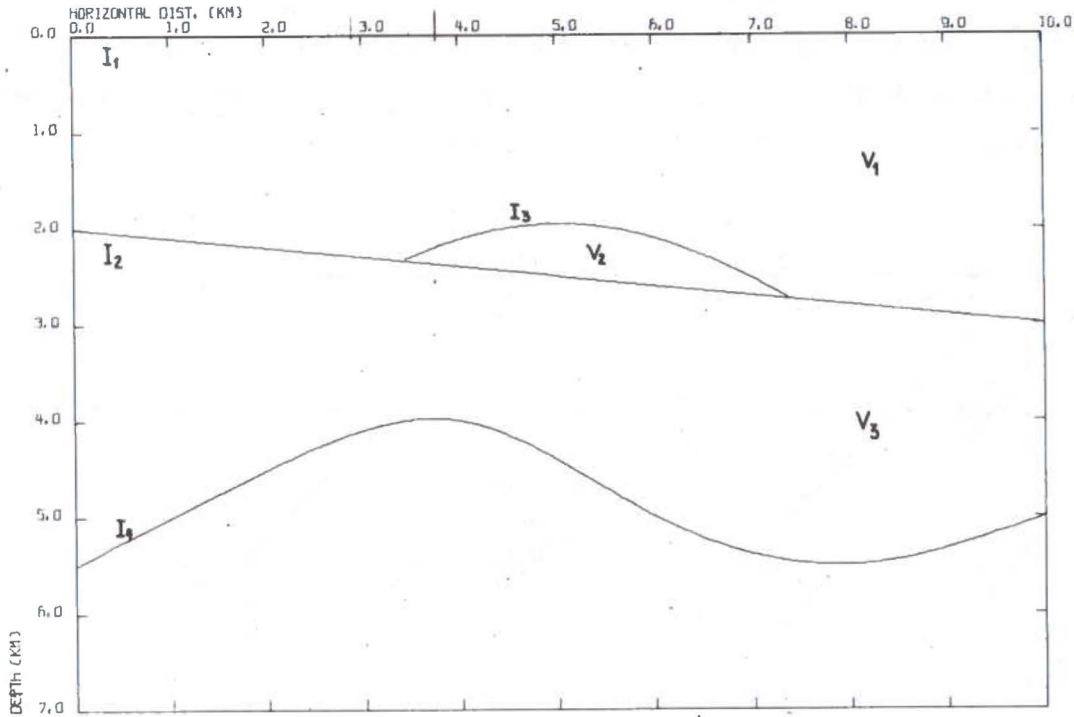


Fig. 9. Cross section of model (plane $y = 5$ km).

6.2 Tracing of normal incidence ray paths (NIP)

We shall now consider a case where we are interested in the normal incidence ray paths from the various interfaces, i.e., the ray paths corresponding to the travel times observed in an ordinary 'stacked' seismic section (see Section 5.2). In this case we will have to search for 'ray foot points' in the start interface which are located in such a way that the normal rays arrive at a predefined 'receiver line' at the surface.

Firstly, we shall place the 'receiver line' parallel to the x-axis (the plane $y = 5$ km), i.e., along the line defining the cross section of Fig. 9. In this plane the model has the property of being '2-dimensional', that is, all normal vectors to the interfaces have y-component equal to 0, since all interfaces are symmetric about this plane. Consequently, all NIP rays arriving at the receiver line will also lie in this plane, which makes it possible to plot them in a 2D plot. In this case the 'search procedure' will be a very simple one, since we will be sure to hit the receiver line

simply by starting in the plane mentioned. Fig. 10 shows the NIP rays in this case, and Fig. 11 shows the corresponding travel time section.

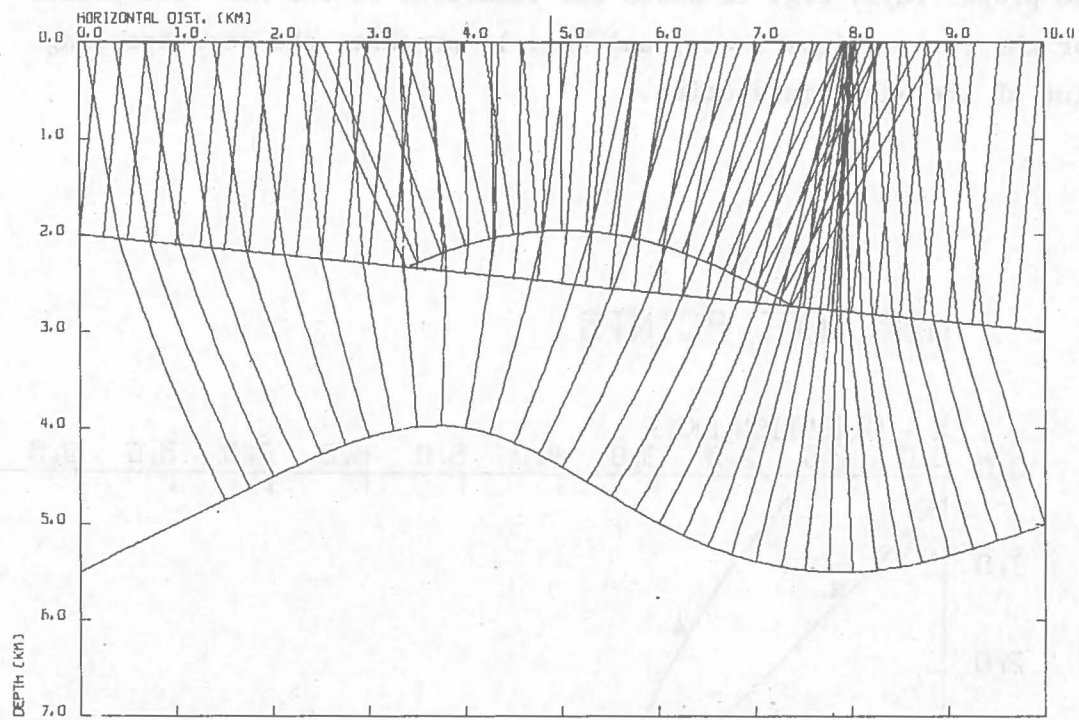


Fig. 10. NIP-rays in the plane $y = 5$ km.

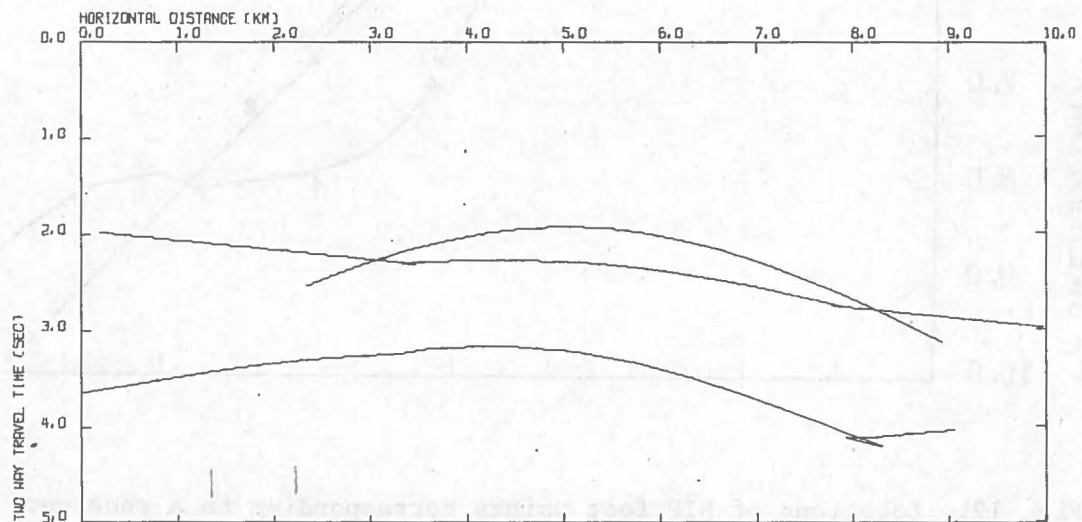


Fig. 11. Normal incidence travel time section corresponding to the rays of Fig. 10.

Turning to a real 3D case, we choose another receiver line, going through the origin and making an angle of 45 deg with the x-axis. Obviously, in this case a real search procedure must be applied in order to obtain the proper rays. Fig. 12 shows the locations of the NIP foot points for the various interfaces, and Fig. 13 presents the corresponding plot of the NIP time section.

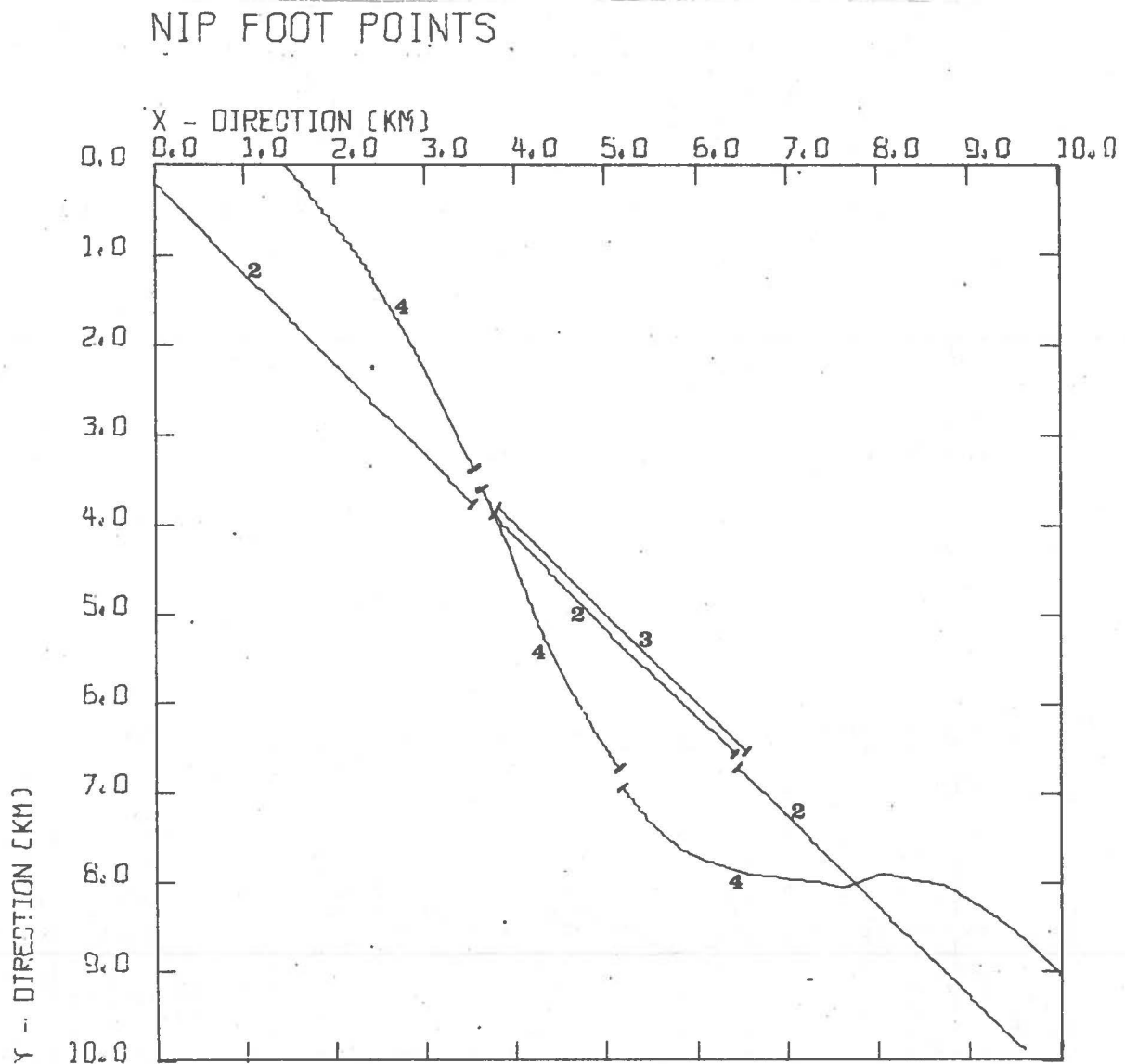


Fig. 12. Locations of NIP foot points corresponding to a receiver line making 45 deg with the x-axis. Reflecting interfaces are indicated.

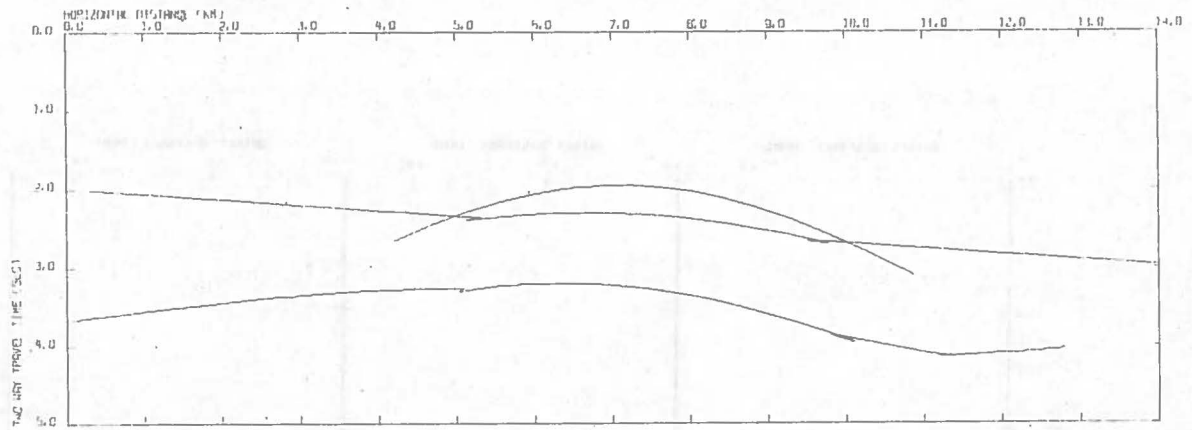


Fig. 13. NIP travel time section corresponding to a receiver line making 45 deg with the x-axis.

6.3 Tracing of non-zero offset ray paths

We shall conclude this section by showing some examples on searching for non-zero offset ray paths arriving at specified receiver lines. Firstly, we shall return to the '2D' cross section on Fig. 9 (receiver line parallel to x-axis in $y = 5$ km). In this case, we know in advance that the y-component of the ray direction unit vector must vanish, and no real 'search' is necessary. Fig. 14 shows some rays emerging from several shot points at the surface, and Fig. 15 shows the corresponding reflection time profiles. The length of each profile is set to 2.5 km.

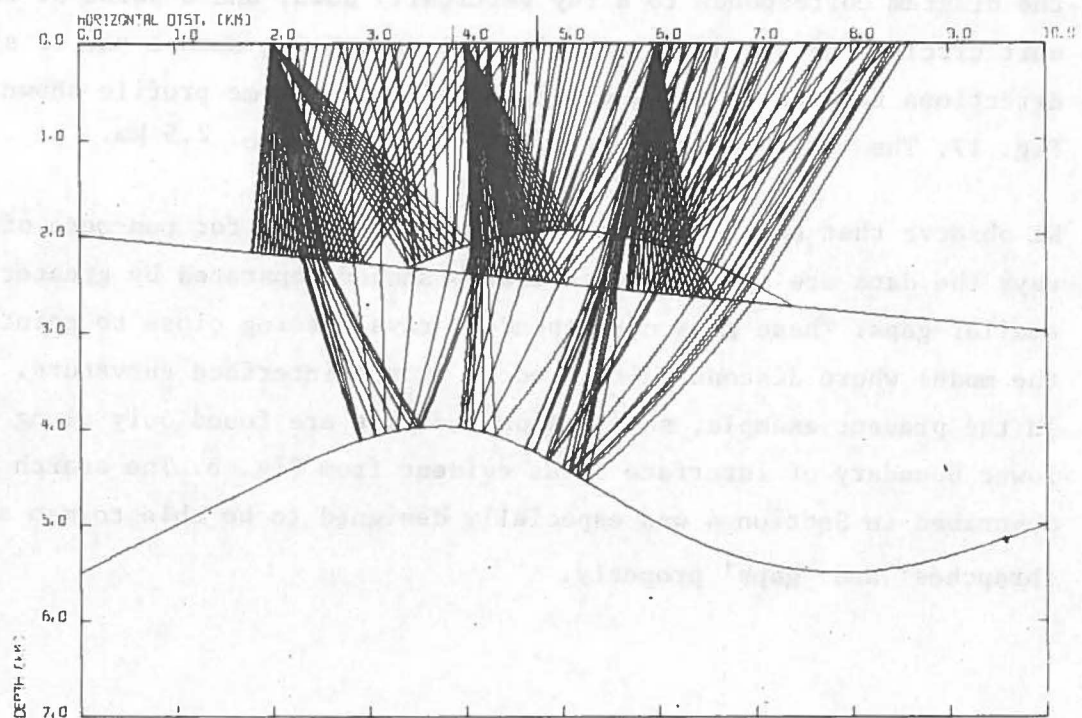


Fig. 14. Non-zero offset rays in the plane $y = 5$ km. Shot points are (2.0, 5.0), (4.0, 5.0), (6.0, 5.0).

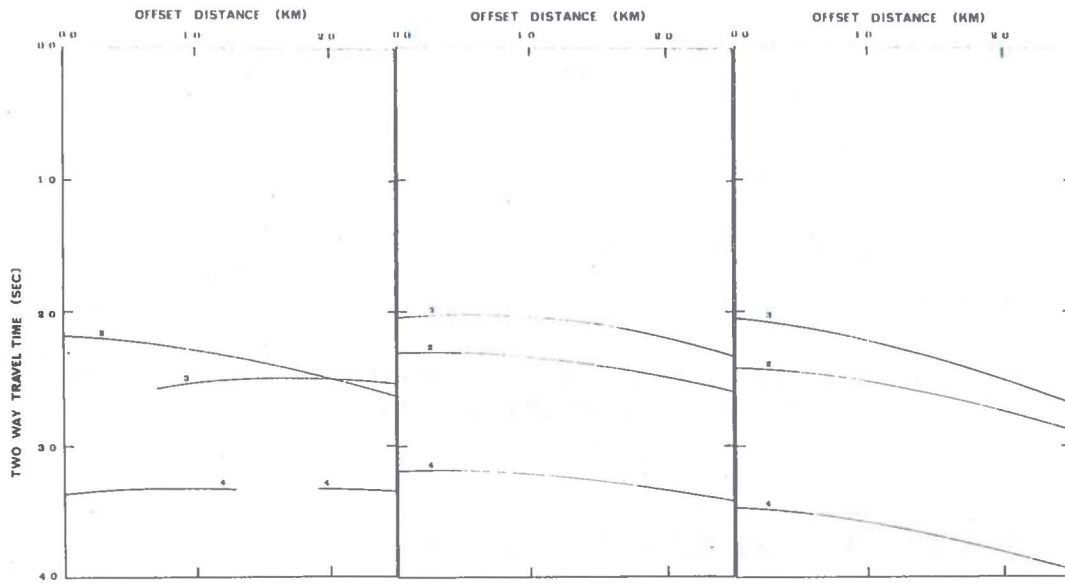


Fig. 15 Reflection time profiles corresponding to shot points (2.0, 5.0), (4.0, 5.0) and (6.0, 5.0), respectively. (Receiver line parallel to x-axis.) Reflecting interfaces are indicated.

Turning again to the 45 deg profile, we search for rays emerging from given shot points and arriving at the given line. Fig. 16 shows a plot of the horizontal components of the ray direction unit vectors we have to start with in order to arrive at this particular line. A point in the center of the diagram corresponds to a ray vertically down, and a point at the unit circle corresponds to a horizontal ray. Using such a set of start directions for the rays, we obtain a reflection time profile shown in Fig. 17. The length of each profile is again set to 2.5 km.

We observe that both for normal incidence rays and for non-zero offset rays the data are split into several branches separated by greater or smaller gaps. These gaps correspond to rays passing close to points in the model where discontinuities occur in the interface curvature.

In the present example, such discontinuities are found only along the lower boundary of interface 3, as evident from Fig. 8. The search procedure described in Section 4 was especially designed to be able to map all these 'branches' and 'gaps' properly.

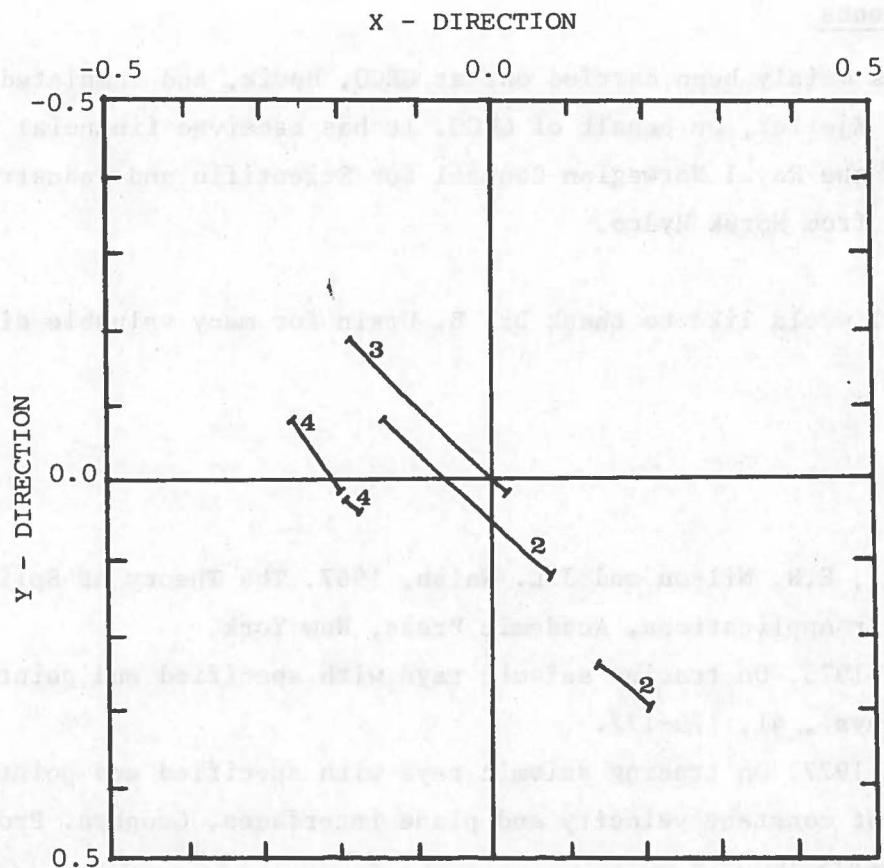


Fig. 16 Diagram showing the horizontal components of the ray direction unit vectors in the start point, for shot no. 3 ($x=6$ km, $y=6$ km). (Receiver line makes 45 deg with the x-axis.) Reflecting interfaces are indicated.

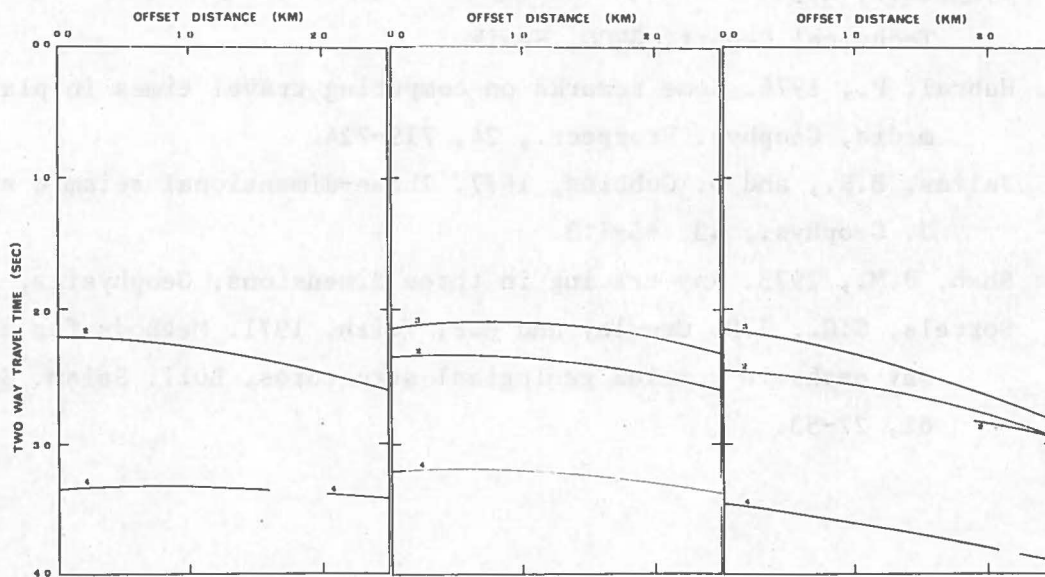


Fig. 17 Reflection time profiles for a receiver line making 45 deg with the x-axis. Shot points are (2.0, 2.0), (4.0, 4.0) and (6.0, 6.0), respectively. Reflecting interfaces are indicated.

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