NORSAR

ROYAL NORWEGIAN COUNCIL FOR SCIENTIFIC AND INDUSTRIAL RESEARCH

Scientific Report No. 1-80/81

SEMIANNUAL TECHNICAL SUMMARY 1 April—30 September 1980

By
Alf Kr. Nilsen (ed.)

Kjeller, November 1980

VI.3 The Effect of a Second-Order Velocity Discontinuity on Elastic Waves Near their Turning Point

For various purposes (i.e., in both studies of the velocity structure and of the seismic source), it is important to understand the effects that certain features of a velocity model have on the elastic wave field. Thus, the effects of a velocity discontinuity and of a velocity gradient have been widely discussed; the effect of a change in the velocity gradient appears to be less understood. In an earth model, second-order velocity discontinuities (i.e., discontinuities in the velocity gradient) may arise due to model parameterization, and their high-frequency effect has been demonstrated in applying geometrical ray theory. It is desirable to smooth this effect since it is an artefact of the model, and this is conveniently done in a WKBJ approximation (Chapman, 1978). However, for some regions of the earth, notably the upper mantle and the base of the mantle, it has sometimes been proposed that rather abrupt changes in velocity gradient occur in a relatively short depth interval. In these cases, the model of one or more second-order discontinuities would still be a simplification but, in analogy to approximating rapid velocity changes by one or more first-order discontinuities, it would be a sensible approximation at relatively long wavelengths. It is the long-wavelength effect that has been studied here. The effect is associated with a change in the curvature of a wavefront across a second-order discontinuity. This change is ignored in the classical WKBJ approximation, but it is described by the extended WKBJ method (the Langer approximation). Following Richards (1976), it is now widely appreciated that the extension of the WKBJ method is most important for long waves near their turning point, consequently the second-order discontinuity is expected to be most effective in the same circumstances. To demonstrate this, generalized wave functions will be used to compute reflection/transmission coefficients. At a second-order discontinuity, the continuity condition for the stress-displacement field reduces to a continuity condition for the wavefield and its vertical derivative. This requires, at least in principle, coupling of up- and down-going waves, but no coupling between P and SV. The reflection/transmission coefficients for P, SV and SH are therefore given by similar expressions

(Upward reflection) $$\frac{A_u^+}{A_d^+} = \frac{U_d^+}{U_u^+} \; \frac{C_d^+ - C_d^-}{C_u^+ + D_d^-}$$

(Downward transmission) $$\frac{A_d^-}{A_d^+} = \frac{U_d^+}{U_d^-} \left(1 - \frac{C_d^- - C_d^+}{C_u^+ + D_d^-}\right)$$

where superscript + and - denote the top and bottom side of the discontinuity, $U_{u/d}$ are up/downgoing wave functions, $A_{u/d}$ the up/downgoing wave coefficients, and $C_{u/d}$ the so-called generalized cosines which are related to vertical derivatives of the waves functions (Richards, 1976). Similar expressions for downward reflection and upward transmission follow from symmetry considerations; for real angle of incidence, downward reflection equals upward reflection in absolute value. In the WKBJ approximation of the wave functions: $C_u^+ = C_d^+ = C_u^- = C_d^- = \cos i$, where i is the angle of incidence, so in this approximation the second-order discontinuity has no effect. However, near a turning point the WKBJ solution is invalid and it has now become almost common practice (Richards, 1976) to extend the approximation by Langer's solution which, among other things, takes into account the difference in curvature of the wavefront on opposite sides of the interface. Fig. VI.3.1 gives an illustration of the effect, in terms of reflection coefficients; obviously, these reflection coefficients can be perhaps surprisingly large for long-period waves near their turning point.

Of course, the model of a single reflector is often an oversimplificatied concept; multiple reflection must be taken into account especially for waves near their turning point. Calculations in a layered model (e.g., reflectivity type of methods) would then account for the effects although these would not be explicitly identified. In fact, one of the motivations for the present study was to explain certain differences between results with the 'classical' reflectivity method (Fuchs and Müller, 1971) and a version of the so-called full wave method which ignores layering. Indeed, introducing layering in the last method, with interfaces coinciding with the second-order discontinuities, satisfactorily removes the discrepancy (Doornbos, 1980). It demonstrates the usefulness of uniformly asymptotic solutions in a piecewise smooth layered model.

D. Doornbos

References

Chapman, C.H., 1978: A new method for computing synthetic seismograms, Geophys. J.R. Astr. Soc., 54, 481-518.

Doornbos, D.J., 1980: The effect of a second-order velocity discontinuity on elastic waves near their turning point, Geophys. J.R. Astr. Soc., in press.

Fuchs, K., and G. Müller, 1971: Computation of synthetic seismograms with the reflectivity method and comparison with observations, Geophys. J.R. Astr. Soc., 23, 417-433.

Richards, P.G., 1976: On the adequacy of plane-wave reflection/transmission coefficients in the analysis of seismic body waves, Bull. Seism. Soc. Am., 66, 701-717.

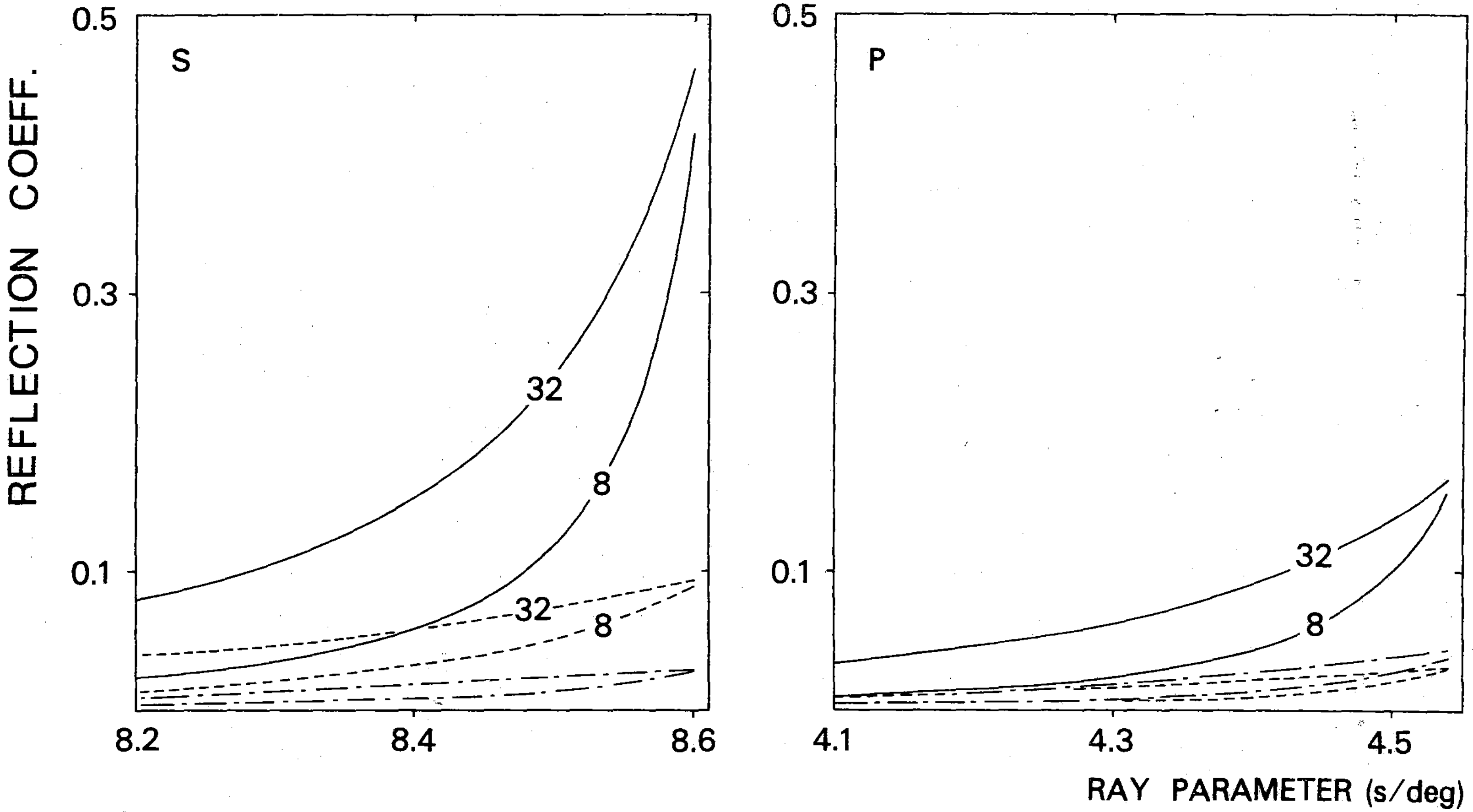

Fig. VI.3.1 Reflection coefficients as a function of ray parameter for P and S waves at a period of 32 and δ s. The interface is a second-order velocity discontinuity at radius 3560.7 km, u_p = 13.661 km·s^{-1}, u_s = 7.218 km·s^{-1}.

________ = du_p^+/dr = -0.001, du_p^-/dr = 0.001g, du_s^+/dr = -0.0004, du_s^-/dr = 0.001 g

-------- = du_p^+/dr = -0.001, du_p^-/dr = -0.001g, du_s^+/dr = -0.0004, du_s^-/dr = -0.001 g

_ - _ - = du_p^+/dr = -0.001, du_p^-/dr = 0 , du_s^+/dr = -0.0004, du_s^-/dr = 0

+ and - refer to the top and bottom side of the interface.