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VI.9 High resolution group velocity analysis AR-method

McCowan (1978) introduces a new high resolution method for measurement of group velocities of seismic surface waves. McCowan's method is based on the instantaneous frequency estimator of Griffiths (1975) which was designed to estimate the frequency content of digital signals with a narrow band and rapidly time varying spectrum. Griffiths and Prieto-Diaz (1977) used this new method, called the adaptive autoregressive (AR) method of data modelling, on seismic data and demonstrated that when conventional methods provide a poor resolution, a high resolution is still achievable by this technique.

The adaptive AR method is most useful for analysis of compact waveforms recorded at regional distances from a seismic source. It will be used for obtaining group velocities in the Middle East using data from the Seismic Research Observatories (SRO) in this region. In this report the AR method is briefly reviewed and a test case on synthetic data presented.

The basic problem is to find group arrival times on a seismogram generated by a near distance seismic source. The group arrival time at a frequency ω_0 is the time of maximum energy at frequencies in the vicinity of ω_0, and is found by spectral estimations.

Denote a set of N equally spaced data points in the time series by

$$x(k),\ k = 0,1,2,\ldots,N-1$$

In the traditional methods (see for example Dziewonski and Hales, 1972), a window (say a Gaussian one) is defined appropriately, convolved with data at regular intervals and each time the amplitude spectrum is calculated by a Fourier transform. The result of this process is then contour plotted and the group arrival times picked from the contours.

In the adaptive AR method, the window (or filter) depends on past values of the signal and is continuously updated as new values become available. The updatation is performed by a simple algorithm after Widrow and Hoff (1960) which is defined as follows:

$$a_\ell(k+1) = a_\ell(k)+\mu\varepsilon(k)\chi(k-\ell) \tag{1}$$

in which the value of the filter coefficient at time (k+1), i.e., $a_\ell(k+1)$, is found from the previous filter and data values. The adaptive step size μ is a parameter which controls the convergence of the scheme and is related to the filter length L and filter learning constant α by:

$$\mu = \frac{\alpha}{L\sigma_\chi^2} \tag{2}$$

where σ_χ^2 is the input power level. The time constant of convergence for the algorithm is given by:

$$\tau = \frac{-1}{\ell n(1-\alpha/L)} \tag{3}$$

$\varepsilon(k)$ in (1) is the filter output at time k and is defined by

$$\varepsilon(k) = \chi(k)-\sum_{\ell=1}^{L} a_\ell(k)\chi(k-\ell) \tag{4}$$

and finally the spectral estimation giving the group arrival time is

$$S(\omega,k) = \left|1-\sum_{\ell=1}^{L} a_\ell(k)e^{-j\omega\ell}\right|^{-2} \tag{5}$$

(see Griffiths and Prieto-Diaz, 1977; McCowan, 1978).

It follows from (3) that the filter constants L and α determine the convergence of the algorithm. The convergence of the adaptive AR method is discussed by Griffiths (1975), Griffiths and Prieto-Diaz (1977), and McCowan (1978) where they suggest that values of between (10-20) and (0.1-2.0) should be selected for L and α respectively.

In summary, the filter parameters L and α must be selected appropriately, filter coefficients at discrete times $k_1, k_2, \ldots, k_n$ calculated using (1) and (4). The spectral estimates are then calculated using (5), contour plotted and group arrival times picked from the contours. If the instrument response is removed from the data prior to filtering, the contours can be calibrated to give the group velocities directly.

The adaptive AR method is capable of tracking instantaneous frequencies (Griffith, 1975). This very important feature can be demonstrated by applying the method to a pure sine wave. Take 100 samples from a 0.05 Hz sine wave (sampled at a rate of one sample per second), and define filter parameters as follows:

Filter length: $L = 12$

Filter learning constant: $\alpha = 0.20$

By using (3) the adaptive time constant will be

$$\tau = \frac{-1}{\ln(1-\alpha/L)} = 59.50 \text{ seconds.}$$

The filter output calculated from (4) is plotted in Fig. VI.9.1a. As expected, the filter output tends to zero at times greater than the adaptive time constant (59.5 seconds), i.e., in this case the present values of the sine wave can be predicted from its past values without any error. The adaptive AR spectral estimates calculated at three different times (20, 60 and 90 seconds) are shown in Fig. VI.9.1b-1d. In Fig. VI.9.1b, the calculation is performed at a time before the adaptive time constant, and there is no spectral peak at 0.05 Hz. In Fig. VI.9.1c, at a time corresponding to a full time constant, a side lobe is still present though its amplitude is more than 12 dB below the main 0.05 Hz signal amplitude. In Fig. VI.9.1d, at a time greater than the full time constant, the side lobes fall more than 40 dB below the main signal level and thus the main signal is easily identified.

In conclusion the adapative AR method is shown to work as expected on synthetic data. The next step is to select seismic events at regional distances from SRO stations and apply this method to find group velocities for many profiles in the Middle East.

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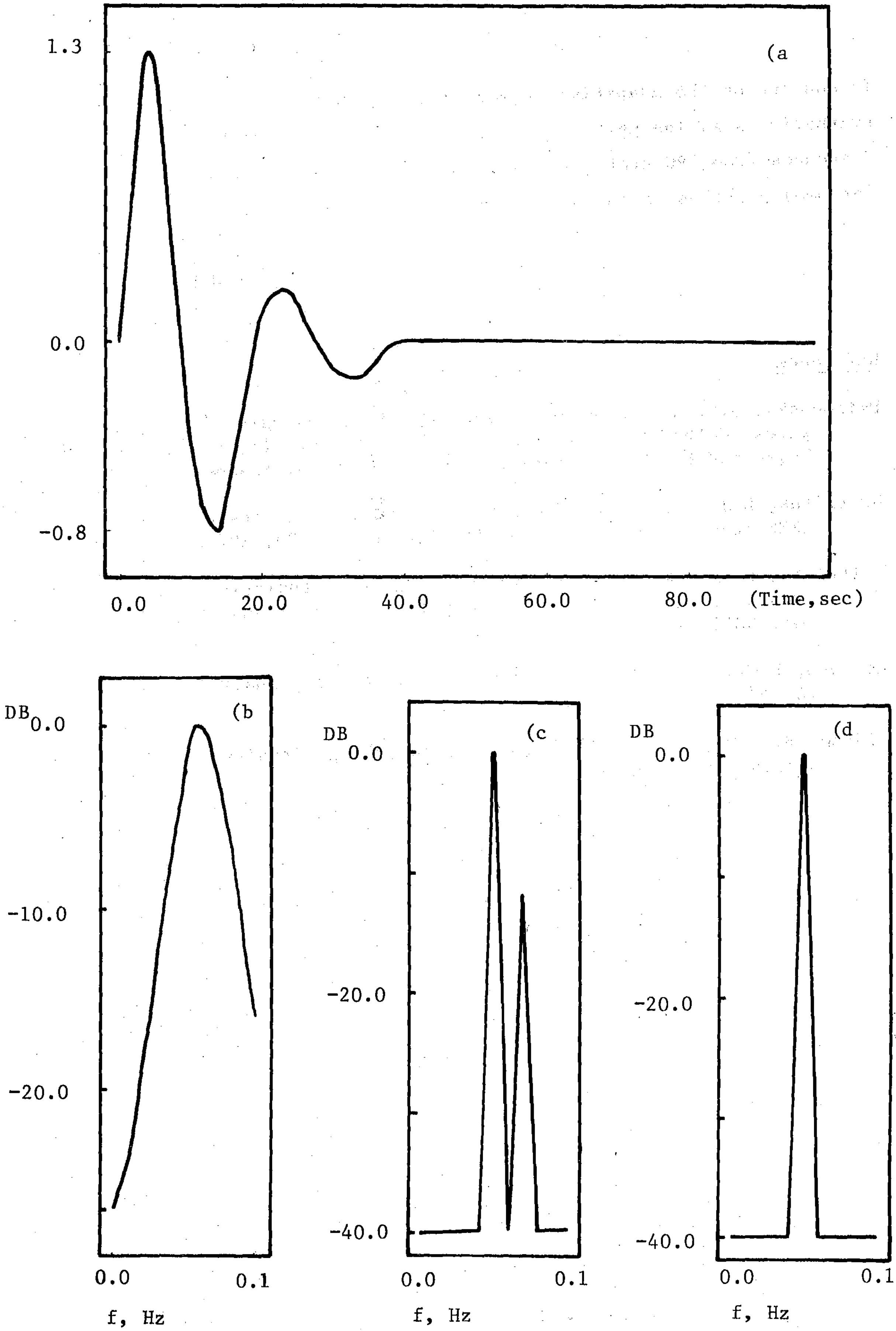

Fig. VI.9.1 Filter output tends to zero at times greater than the adaptive time constant (59.5 sec) (a). Adaptive spectral estimates are calculated at 20 sec (b), 60 sec (c), and 90 sec (d). See text.