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VI. 8 Dynamic ray-tracing in complex three-dimensional models

In the last years, 3-D seismic modelling has become a rapidly growing branch of modern geophysical research. Within this area a very important tool is the seismic ray-tracing methods, which have recently been extended to include even dynamic properties of the wave field.

We have developed a rather general 3-D ray-tracing system suitable for application to various kinds of seismic modelling problems. (For details see Gjøystdal, 1978A,B; Gjøystdal, 1979; Gjøystda1 \& Ursin, 1981; Gjøystdal et al, 1981; Reinhardsen, 1981.)

One of the major problems considered is how to represent a 3-D geological model mathematically. The various seismic reflectors are divided into a system of bicubic spline surfaces. A special kind of 'logic' has been developed in order to obtain a proper connection between the various spline interfaces, permitting an unambiguous tracing of any ray through the model, provided the ray has specified direction in the start point. The layers between the interfaces may have any velocity variation, provided the velocity function is continuous and has continuous lst and 2nd derivatives everywhere in each layer.

Particularly in the last 5 years there have been a number of works on the problem of extending the ray-tracing procedures to include parameters in addition to those just mentioned. For example, Hubral and Krey have published several papers on wavefront curvature calculations, and last year Cerveny and Hron presented an excellent theoretical work on the ray series method and dynamic ray-tracing in 3-D inhomogeneous media. (For reference, see f.ex. Hubral and Krey, 1980; and Cerveny and Hron, 1980.) We have included these methods in a $3-\mathrm{D}$ ray-tracing system. A new dimension is introduced into such a dynamic ray-tracing system in that we are now able to calculate wavefront curvatures and amplitudes at any point of a ray (see Fig. VI.8.1). The 'dynamic' procedures are able to make proper use of the parameters of the medium sampled by the ray, such as velocity gradients, interface curvatures, etc., which were just ignored in 'conventional' ray-tracing.

In mathematical terms, the wavefront is represented by a 2 by 2 wavefront curvature matrix, which essentially contains the wavefront curvatures in two principal directions perpendicular to each other. There now exist theoretical procedures for calculation of the change in the wavefront curvature matrix on refraction or reflection at an interface. In order to calculate the change in the wavefront curvature matrix and amplitude coefficients when tracing the ray through a continuous part of the medium, we have to solve a set of differential equations. These are non-1inear equations that must generally be solved by numerical approximations. For general velocity media; we have used the so-called Admas P-E-C-E method in order to solve the equations (Predict-Evaluate-Correct-Evaluate). For media with constant velocity or constant velocity gradient, we can obtain analytical solutions of the equations, of course making the process considerably more efficient.

We have developed a special search procedure in order to find ray paths connecting a given source- and receiver-position in a 3-D model. It is based on the 'shooting method', that is, we are starting in the shot point with a certain inftial direction of the ray, and the ray is traced through the model until a specified 'receiver interface' has been reached. The initial ray direction is then updated and a second ray is traced through the model. The procedure is then repeated until the ray arrives sufficiently close to the specified receiver point. The procedure is described in detail in Gjøystdal (1978A, B) ; here we shall restrict ourselves to stating some basic properties of the method. The procedure takes advantage of a 'receiver line' running through the receiver point. The search constitutes a 'curve crawling process' along this line, using gradient calculations for updating the ray direction at each iteration step. The procedure is especially efficient when a number of receivers are distributed along a line (or continuous curve), as is usually the case in geophysical exploration. In addition, the procedure is designed to pick up the various branches of the travel time function if such branches exist, thus being able to determine all ray paths connecting source and receiver in a complex 3-D model.

It should also be mentioned that during this search procedure, no dynamic parameters (i.e., wavefront curvature, amplitude, etc.) are calculated. Such calculations are carried out only for the resulting ray paths. These rays are traced through the medium once more, and the necessary integrations are performed in order to determine the parameters wanted.

A simple example is shown in Fig. VI.8.2. Fig. VI.8.2a shows a 3-D model, consisting of 5 interfaces. The seismic velocities in each layer are generally varying continuously with space coordinates. Fig. VI. 8.2 b shows a vertical cross section through the model along the line $A-B$ in Fig. VI.8.2a, together with normal incidence rays for this line projected into the cross section. Because of the 3-D nature of the problem the ray paths will generally not intersect the interfaces in this cross section. Fig. VI.8.2c shows the corresponding zero offset seismic section. Note that each amplitude has been scaled proportional to the travel time, in order to compensate for the large dynamic range of the various arrivals.

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## 'DYNAMIC' RAY-TRACING

## CALCULATION OF

- RAY PATH (COORDINATES AND DIRECTION)
- travel time
- TRAVEL DISTANCE
- WAVEFRONT CURVATURE
- AMPLITUDE


Fig. VI. 8.1 Dynamic ray-tracing in 30 models - schematic illustration.

a)

Fig. VI.8.2 (a) Three-dimensional plot of the synthetic model. Vertical scale is exaggerated. Horizon extension is $10 \times 10 \mathrm{~km}$; vertical extension is 4 km .



FIg. VI. 82 (b) Vertical cross section of the model along the shot/recelver ine. Horizontal coordinate 0.5 km is at position (2.,2.).
(c) Synthetic seismograms for a general 3-D model with nonlinear velocities. Horizontal coordinates along shot/receiver line.

