NORSAR

ROYAL NORWEGIAN COUNCIL FOR SCIENTIFIC AND INDUSTRIAL RESEARCH

Scientific Report No. 1-82/83

SEMIANNUAL TECHNICAL SUMMARY

1 April - 30 September 1982

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Kjeller, January 1983

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VI.4 Seismic energy and stress drop from moment tensor analysis

In the previous semiannual summary we discussed suitable ways of extrapolating low-frequency approximations of the source spectrum to higher frequencies. A typical low-frequency approximation is the source representation by moments of low degree, and a suitable extrapolation, at least for practical purposes, was suggested to be given by a Gaussian spectrum. The ω-square model was given as an alternative. It should be noted that each of these spectral source models provides an estimate of seismic energy and apparent stress, and the second degree moments in each model can be interpreted in terms of source dimensions from which an estimate of static stress drop may be inferred. A number of practical procedures for estimating the parameters in these source models is under development; at this stage the restriction is that the source be not too shallow. This is due to our present treatment of Green's functions in the inversion procedure. Here we summarize the procedure and give preliminary results for a deep event as an example. A more detailed treatment is given in Doornbos (1982).

For a bounded source function with zero degree moment tensor $\underline{\underline{M}}$, an expression for the total radiated seismic energy is

$$E_s = \frac{1}{16\pi^2\rho} \int_\Omega d\Omega \left[\alpha^{-5} B^2(\underline{\zeta}_p) E(\underline{\zeta}_p) (\gamma_j \gamma_k M_{jk})^2 \right.$$

$$\left. + \beta^{-5} B^2(\underline{\zeta}_s) E(\underline{\zeta}_s) \{ \gamma_j \gamma_k M_{j\ell} M_{k\ell} - (\gamma_j \gamma_k M_{jk})^2 \} \right] \qquad (1)$$

where α and β are P and S velocities, $\underline{\zeta}_p$ and $\underline{\zeta}_s$ are the associated slowness vectors, γ_i are direction cosines, and we have introduced the energy E and spectral bandwidth B of a pulse by

$$E(\underline{\zeta}) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} |F(\underline{\zeta},\omega)|^2 , \quad B^2(\underline{\zeta}) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \omega^2 |F(\underline{\zeta},\omega)|^2 / E(\underline{\zeta}) \qquad (2)$$

For the Gaussian and ω-square models with second degree moment tensor $\hat{\underline{\underline{F}}}_{(2)}$:

$$B^2(\underline{\zeta})E(\underline{\zeta}) = \chi(\underline{\zeta}^T\hat{\underline{\underline{F}}}_{(2)}\underline{\zeta})^{-3/2} \qquad (2)$$

$$\chi = 1/4\ \pi^{-\frac{1}{2}} \qquad \text{(Gaussian)}$$

$$\chi = 1/\sqrt{2} \qquad \text{(ω-square)}$$

In general the integral over solid angle in equation (1) should be evaluated numerically. Only for a point source is there an analytic solution. It is more convenient to rewrite the quadratic form in equation (2) in the principal axes system of $\hat{F}_{\ell m}$, the spatial part of $\hat{\underline{\underline{F}}}_{(2)}$. Let $\underline{p}_i$ be the unit eigenvectors of $\hat{F}_{\ell m}$ and $\lambda_i{}^2$ the associated (positive) eigenvalues. Then:

$$\underline{\zeta}^T\hat{\underline{\underline{F}}}_{(2)}\underline{\zeta} = \frac{\lambda_i{}^2}{c^2}(\underline{\gamma}\cdot\underline{p}_i)^2 + \frac{2}{c}\hat{F}_{p_1,\tau}(\underline{\gamma}\cdot\underline{p}_1) + \hat{F}_{\tau\tau} \qquad (3)$$

where c is the wave velocity, and rupture is supposed to extend along the major principal axis. An interpretation of the spatial moments $\lambda_i{}^2$ and temporal moment $\hat{F}_{\tau\tau}$ in terms of an equivalent uniform source region is

$$V_u = \frac{20}{3}\pi\ \lambda_1\lambda_2\lambda_3\ , \qquad S_u = 4\pi\ \lambda_1\lambda_2\ ,$$

$$T_u = 2\sqrt{3}\ \hat{F}_{\tau\tau}{}^{\frac{1}{2}} \qquad (4)$$

where V_u is a volume, S_u is a surface appropriate for a plane fault, and T_u is a time length. A stochastic interpretation of the moments leads to slightly different constants in the expressions for V and T.

If Green's functions are determined by only one asymptotic wave, an estimate of the 'travel time residual' for each station and phase, ΔT_i, can be obtained by standard travel time analysis, and determining the first degree moment tensor $\underline{F}_{(1)}$ from the linear system

$$\Delta T_i = \underline{\zeta}_i^T \underline{F}_{(1)} \tag{5}$$

would then amount to the usual procedure of estimating source location. This will not be further pursued here. Simple Green's functions in the above sense can be decomposed, and the system of equations for determining the moments of degree zero and two becomes

$$U_i(\underline{x},\omega) = A_i G_i(\underline{\xi}_o,\underline{x},\omega) \exp(-\tfrac{1}{2}\omega^2 B_i - i\omega\tau_o) \tag{6}$$

with

$$A_i = (s_j \zeta_k)_i M_{jk} \tag{7}$$

$$B_i = \underline{\zeta}_i^T \hat{\underline{\underline{F}}}_{(2)} \underline{\zeta}_i \tag{8}$$

where $\underline{s}$ is the unit displacement vector of the wave in the source reference point $\underline{\xi}_o$. Although equations (7) and (8) are both linear systems, it is recommended to estimate $\hat{\underline{\underline{F}}}_{(2)}$ by nonlinear inversion using equation (6) directly, and to use physically plausible constraints in the procedure.

As an example of the procedure, we have inverted long- and short-period SRO data from a deep-focus event. The observations are displayed in figure 1, and pertinent results for three cases (point source, circular source and general ellipsoidal source) are given in table 1. In the context of a shear dislocation model, the apparent stress $\eta\bar{\sigma}$ may be obtained from scalar moment M and seismic energy E_s, and the stress drop $\Delta\sigma$ may be obtained from scalar moment and fault shape and surface area (e.g., Aki, 1972). The stress drops listed in table 1 were simply for a circular fault shape. Despite the rather different constraints the results of cases (a) and (c) are reasonably close, suggesting that reasonable estimates of seismic energy and stress drop can be obtained without knowing the fault geometry and rupture history in detail. Nevertheless a number of practical problems remains and figure 2 serves to illustrate some of them, namely, the possible effects of limited bandwidth of the digital SRO system, and frequency dependence of Q. These problems were already identified previously.

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References

Aki, K. (1972): Earthquake mechanism. Tectonophysics, 13, 423-446.

Doornbos, D.J. (1982): Seismic source spectra and moment tensors. Phys. Earth Planet. Int., in press.

	M	N	λ_1^2	λ_2^2	λ_3^2	$\lvert\hat{F}_{\ell\tau}\rvert$	$\hat{F}_{\tau\tau}$	RMS error	
	(10^{25} dyne·cm)			(km^2)		(km·s)	(s^2)		
(a)	1.92 (±0.06)	0.17 (±0.05)	16.8 (±0.9)	15.7 (±1.3)	1.5 (±3.5)	2.0 (±0.1)	0.45 (±0.06)	44	$E_s = 2.4 \times 10^{19}$ erg $S = 204$ km^2 $\Delta\sigma$=16 bar $\eta\bar{\sigma}$=1.5 bar
(b)	1.92 (±0.06)	0.17 (±0.05)	0	0	0	0	0.33 (±0.02)	148	$E_s = 4.4 \times 10^{19}$ erg
(c)	1.92 (±0.06)	0.17 (±0.05)	9.4 (±1.2)	9.4 (±1.2)	0	0	0.25 (±0.02)	155	$E_s = 3.2 \times 10^{19}$ erg $S = 118$ km^2 $\Delta\sigma$=36 bar $\eta\bar{\sigma}$=2.0 bar

Table VI.4.1 Fiji Islands event with PDE origin time 1980, June 17, 8 hr, 42 m, 56.9 s, location 20.175°S, 178.443°W, depth 580 km, m_b 5.6. M and N are scalar moments of major and minor double couple, $\hat{\lambda}_i^2$ are positive eigenvalues of the spatial moment tensor, $\hat{F}_{\tau\tau}$ is temporal moment. $\lvert\hat{F}_{\ell\tau}\rvert$ is length of spatial-temporal moment, E_s is total radiated seismic energy, S is fault surface area, $\Delta\sigma$ is stress drop, $\eta\bar{\sigma}$ is apparent stress. Standard deviations in parentheses. Results for (a) general model, (b) point source, (c) prescribed rupture on circular fault.

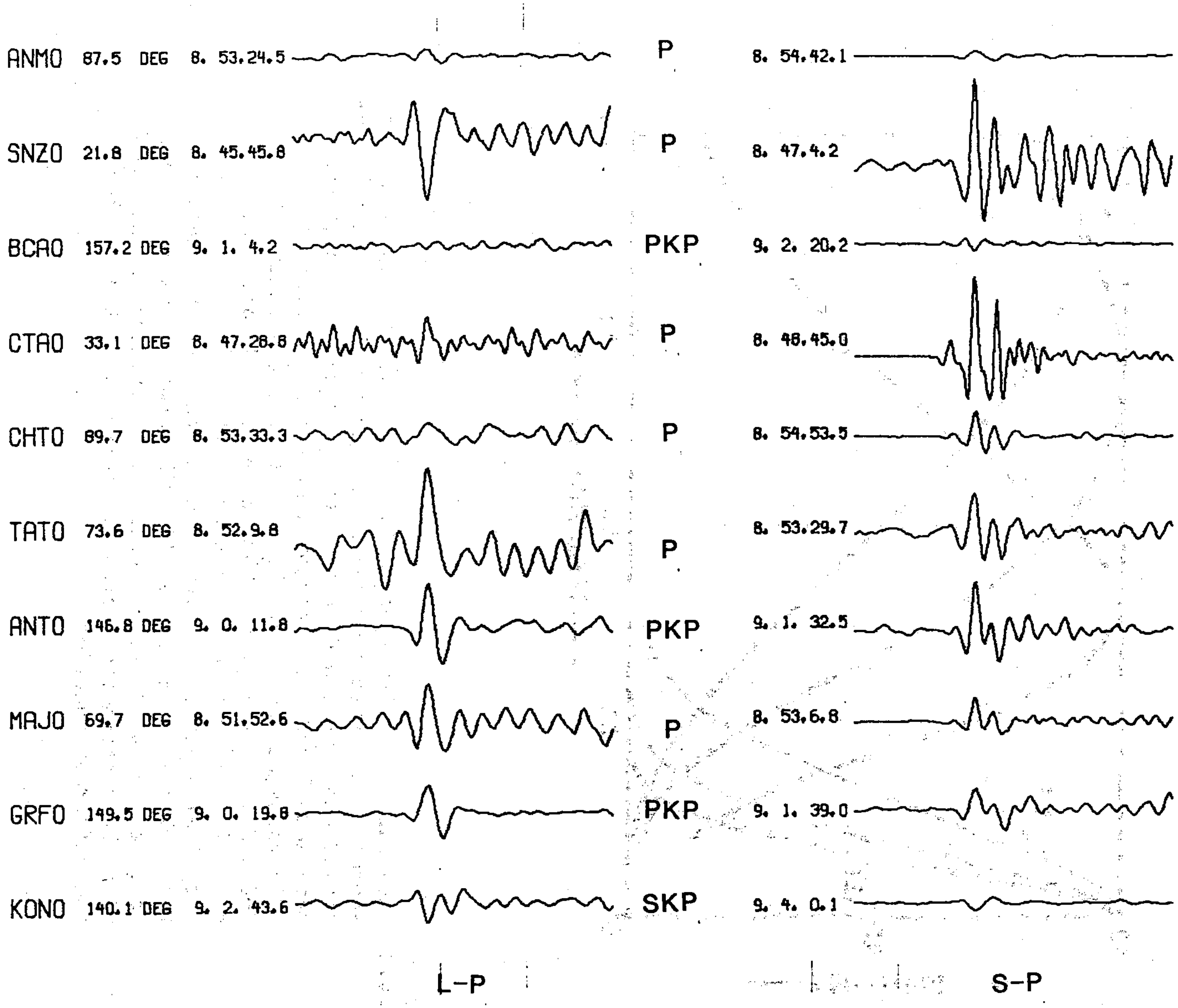

Fig. VI.4.1 SRO and ASRO records from Fiji Islands events (see Table VI.4.1 for details) with vertical components of P, PKP or SKP. Different amplitude scale for long- and short-period sections. Long-period record length is 4 minutes, short-period record length is 12 seconds.

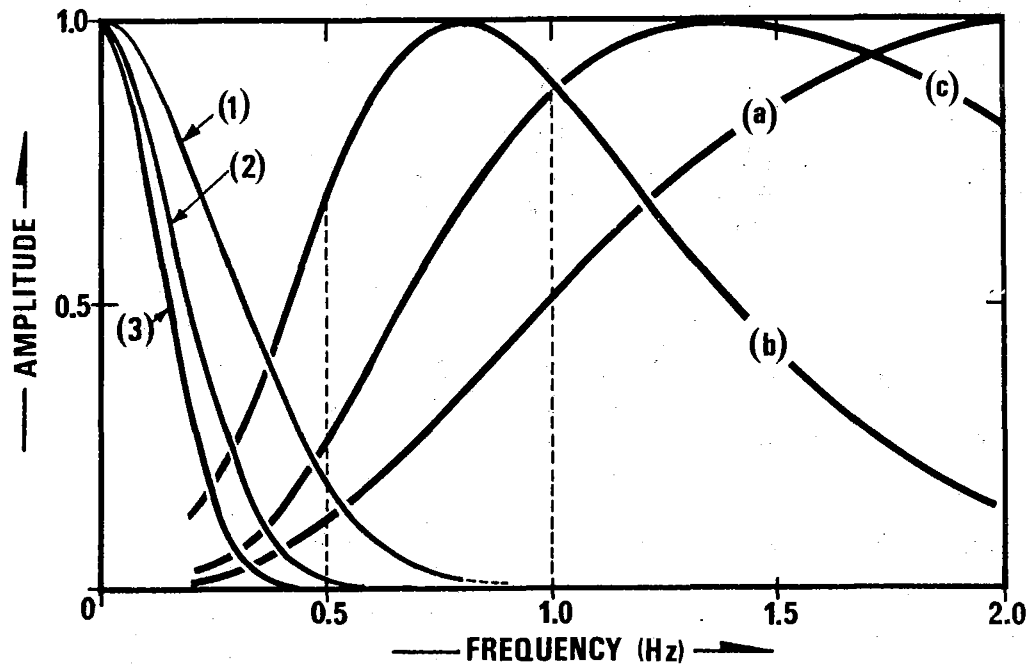

Fig. VI.4.2 Gaussian excitation spectra (1), (2), (3), for spatial point source with finite temporal moment obtained by fitting long-period amplitudes and short-period energy. (1): short-period bandwidth 0.5-2.0 Hz, temporal moment $F_{\tau\tau} = 0.33\ s^2$; (2): bandwidth 0-1.0 Hz, $F_{\tau\tau} = 0.82\ s^2$; (3) bandwidth 0-0.5 Hz, $F_{\tau\tau} = 1.47\ s^2$. The curve (a) indicates the digital short-period SRO response, (b) and (c) indicate a typical Green's function in a standard earth model, for P at teleseismic distance. (b) PREM Q-model (used with the long-period data), (c) Q-model from Archambeau et al (used with the short-period data). Different amplitude scales for the different functions.