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VI.2 Spectral bandwidth, pulse width and moments in source analysis

An observational parameter often employed in source analysis is the spectral bandwidth (usually inferred from a corner frequency in the spectrum); for example radiated energy is proportional to a weighted average of the spectral bandwidth squared. On the other hand, a measure of source size is given by the time domain pulse width (more precisely, the second central moment of the pulse). Although these two parameters are simply related for any point on the focal sphere, the effect of averaging can be quite different.

It is possible to discuss this effect in the context of a moment tensor representation of the source. If the temporal derivative of stress glut is approximated (Doornbos, 1982):

$$\dot{m}_{jk}(\underline{\xi},\tau) = M_{jk}\ f(\underline{\xi},\tau) \tag{1}$$

then the scalar function $f(\underline{\xi},\tau)$ can be expanded in moments. If we choose as the reference point the 'center of gravity' or 'centroid' $(\hat{\underline{\xi}}_o,\tau_o)$ of $f(\underline{\xi},\tau)$, then the expansion is in terms of the central moments. We cancel the observationally troublesome phase effect by relating the moments to the amplitude density spectrum of a (normalized) pulse $f(\tau)$:

$$|F(\omega)|^2 = 1-\omega^2\ F_{(2)} + \frac{1}{12}\ \omega^4\ (\hat{F}_{(4)} + 3\ \hat{F}^2_{(2)}) + \dots \tag{2}$$

and directivity is included in the model by replacing

$$f(\tau) \text{ by } f(\tau,\underline{\xi}),\ \hat{F}_{(2)} \text{ by } \underline{\zeta}^T\hat{\underline{\underline{F}}}_{(2)}\underline{\zeta}\ ,\ \text{etc.}$$

Here $\underline{\zeta}$ is a generalized slowness vector as defined in Doornbos (1982). For practical purposes it is necessary to reduce the number of parameters, and this is done by means of a suitable extrapolation of the spectrum based on the second central moments. The Gaussian and ω-square models are representative examples.

The corner frequency has been conventionally determined as an average over the focal sphere. Similarly we can form the averaged spectral bandwidth for P and S waves:

$$\bar{B}_c = \frac{1}{4\pi} \int_\Omega B(\underline{\zeta}_c)\, d\Omega \tag{3}$$

where c is the P or S velocity. The result will depend on the directivity of $\mathbb{f}(\underline{\xi},\tau)$, in particular on the effect of rupture velocity. In contrast, the averaged pulse width squared is related to

$$\bar{F}_c = \frac{1}{4\pi} \int_\Omega \underline{\zeta}_c^T \hat{F}_{(2)} \underline{\zeta}_c\, d\Omega = \frac{1}{3c^2} (\hat{F}_{xx} + \hat{F}_{yy} + \hat{F}_{zz}) + \hat{F}_{tt} \tag{4}$$

where $\hat{F}_{ii}$ are the diagonal components of $\hat{\underline{\underline{F}}}_{(2)}$. This result does not explicitly involve rupture velocity. As a measure of pulse width we will use $\bar{D}_c = \chi\, \bar{F}_c^{\frac{1}{2}}$, with χ a (model dependent) constant. Usually an inverse square root relation between B and $\hat{F}_{(2)}$ exists for any point on the focal sphere, but for sources with strong directivity effects (e.g., Haskell type of models), the result of averaging procedures (3) and (4) can be quite different. This is illustrated by comparing Figs. VI.2.1 and VI.2.2, which give results for moment tensor approximations to a Haskell bidirectional model with aspect ratio 0.4 and to a circular model, respectively. The 'corner frequency shift' in terms of $\bar{B}_\alpha/\bar{B}_\beta$ is also illustrated in these figures (c.f. Hanks, 1981). $\bar{F}_c$ can be related to source finiteness, and $\bar{B}_c$ to the dominant frequency range for energy radiation.

D.J. Doornbos

References

Doornbos, D.J., 1982: Seismic source spectra and moment tensors, Phys. Earth Planet. Inter. 30, 214-227.

Hanks, T.C., 1981: The corner frequency shift, earthquake source models, and Q, Bull. Seism. Soc. Am. 71, 597-612.

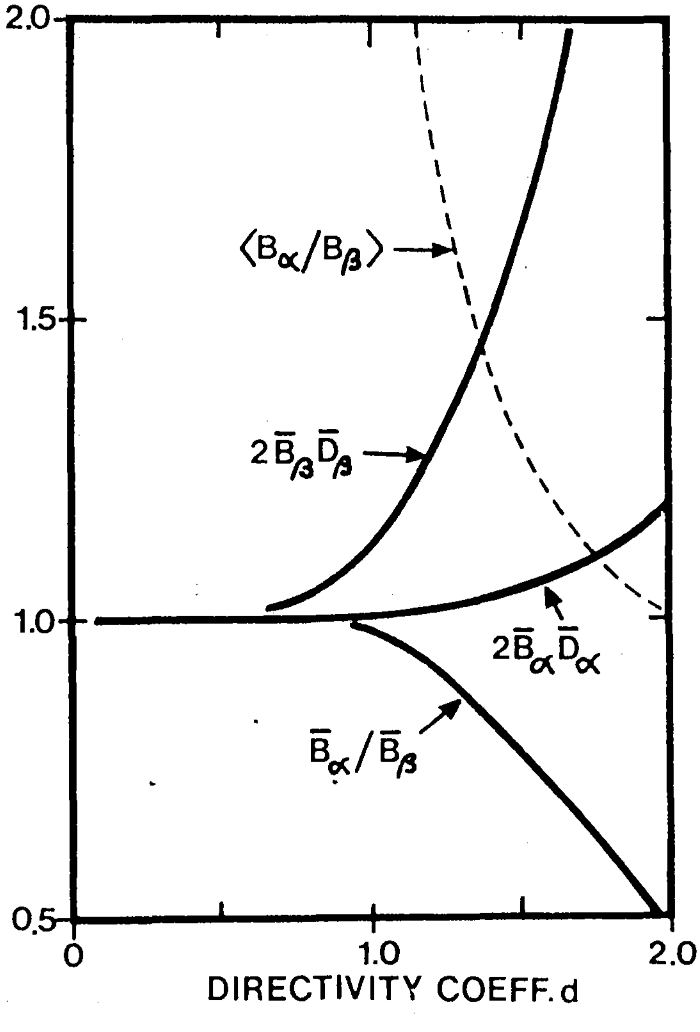

Fig. VI.2.1 Results for averaged spectral bandwidth and pulse width $\bar{B}_c$, $\bar{D}_c$, c the P or S velocity. A Gaussian spectral model has been used in the moment tensor approximation to the Haskell bi-directional model with aspect ratio 0.4. The directivity coefficient $d = \hat{F}^{\frac{1}{2}}_{xx}/(\beta\ \hat{F}^{\frac{1}{2}}_{tt})$. For any point on the focal sphere $2\ B_c\ D_c = 1$, but the averaged values give in general $2\ \bar{B}_c\ \bar{D}_c > 1$. The dotted curve denoted $\langle B_\alpha/B_\beta \rangle$ gives the ratio of areas of the focal sphere where $B_\alpha > B_\beta$ and $B_\alpha < B_\beta$, respectively.

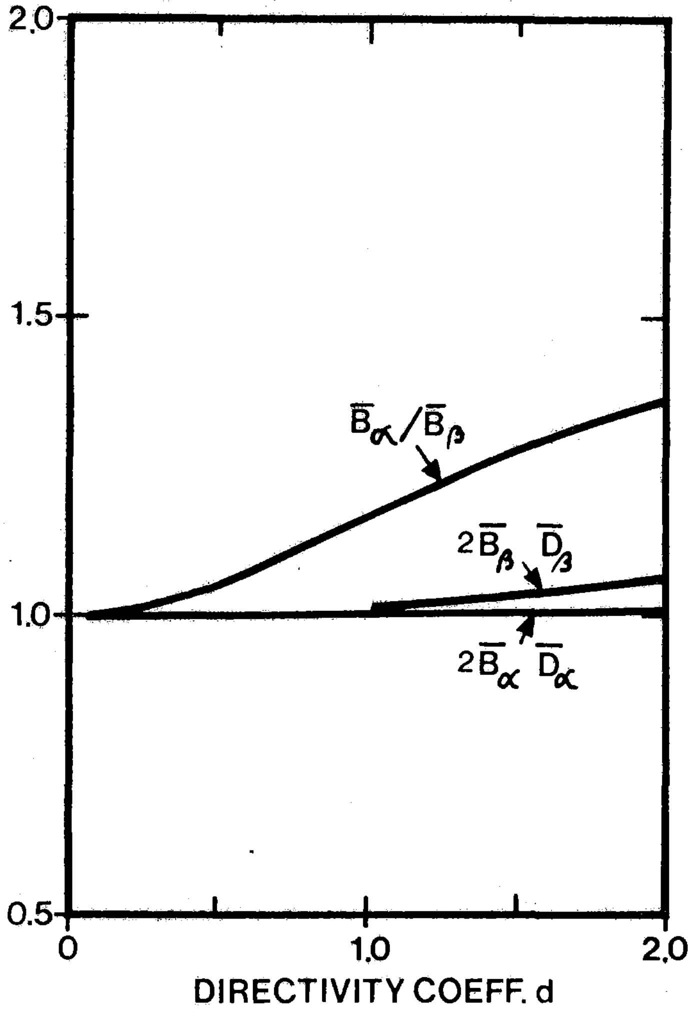

Fig. VI.2.2 Results for averaged spectral bandwidth and pulse width B_c, D_c, in a moment tensor approximation to a circular model. Other details as in Fig. VI.2.1.