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VI.5. Weighted beamforming in a real time environment

As is well known, the extent of signal and noise correlation between sensors in an array might significantly affect its performance in terms of suppressing the ambient noise while retaining signal integrity. The importance of signal and noise correlations as a function of sensor separations and frequency band has recently been demonstrated by Mykkeltveit et al (1983) in a scheme for optimizing array configurations. A rather obvious result here is that once the array becomes operational (configuration fixed) its performance is much dependent on dominant signal frequency. For example, a miniarray designed for optimum detection capabilities for signals from local and regional events, say in the range 5-15 Hz, would be far less efficient for teleseismic signals in the frequency range 1-3 Hz. From a general seismological point of view, the preference is for array operation which is not strongly peaked as a function of frequency. In practice this requires that a flexible weighting scheme is introduced as part of the array on-line operation, where we try to capitalize on the information contained in the noise covariance matrix. The weighting technique used, to be briefly described in the following, is very similar to the optimum processing schemes developed by Capon, Lacoss and others (e.g., see Lacoss, 1974).

Signal/noise modelling and optimum weight estimation

Many weighting/filtering schemes have been developed for multichannel noise suppression. The best known class here is the Wiener filters which utilize the information contained in the autocovariance matrix, while in our scheme the subset hereof, the covariance matrix, is in focus.

Model I - Signals identical:

$y_i = s + n_i$; s = signal, n_i = noise i-th sensor

$E(n_i n_j) = T_{ij}$

$\Sigma = (T_{ij})$ = noise covariance matrix

Standard (unweighted) beams:

$$\hat{s} = \sum_{i=1}^{k} y_i = ks + \Sigma n_i \qquad \ell_{1,k} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$

$$\hat{s} = k^2 s^2 + \ell' \Sigma \ell$$

$$\text{Gain}^2 = \frac{k^2}{\ell' \Sigma \ell} = 10 \cdot \log \left(\frac{k^2}{\ell' \Sigma \ell} \right) \text{ dB}$$

Weighted beam:

$$\hat{s} = \sum_{i=1}^{k} w_i y_i = s \, \Sigma \, w_i + \Sigma \, w_i n_i \qquad w_{ik} = \begin{bmatrix} w_1 \\ \cdot \\ \cdot \\ \cdot \\ w_k \end{bmatrix}$$

$$\text{Gain}^2 = \frac{(\ell' w)^2}{w' \Sigma w}$$

By derivation of the gain function we find:

$$\max(\text{Gain}^2) \text{ for } w \sim \Sigma^{-1} \ell$$

In practice we want to use a normalized weight function, and this introduces the w_{norm}-weights:

$$w_{norm} = \ell' w$$

$$w_{use} = w / w_{norm}$$

Implicit in this estimation scheme is that $\Sigma w_i = 1$. Important, in order to avoid negative weights, the w_i elements in the above gain function are replaced by $w_i = x_i^2$. For $k < 20$ the minimizing of the gain function (is negative counterpart) can easily be performed directly without derivatives, etc.

Model II - Signals not perfectly correlated

In the case the gain would be:

$$\text{Gain}^2 = \frac{w'\Omega w}{w'\Sigma w}$$

with Ω being the signal covariance function.

Again, we can show that:

$$\text{max Gain for } (\Omega - \lambda\Sigma)\, w = 0$$

where λ is the largest eigenvalue and w the corresponding eigenvector. We may also extend the model to include a priori knowledge of signal amplitudes (e.g., see Christoffersson and Husebye, 1974). Note that significant gain in beamforming is obtainable when sensor noise correlation is significant. Restricting weights to be positive only essentially amounts to 0/1 weighting.

Practical considerations

As in all other noise suppression schemes, the critical factor is noise stationarity, in this particular case the time/space stability of the noise covariance estimate in use. Parameters of importance here are:

- IW = length of data window for covariance estimate (2.5-10 sec); in average most stable results for IW = 10 sec or 400 samples
- INCR = covariance matrix updating rate; typical values 1.25-2.50 sec; in average most stable results for INCR = 2.5 sec or 100 samples
- ALFA = covariance matrix smoothing parameter; similar functioning of the STA/LTA sliding windows as used for signal detectors; here defined as ALFA = 1. - EXP(ALOG(1.-PRO)/FIW) with PRO = 0.5 and FIW = IW + 1

Now, parameters of importance in judging the performance of the above weighting scheme are as follows:

- TN = theoretical gain permitting negative weights tied to the covariance matrix only; no real data used
- ON = optimum gain (neg. weights permitted) as measured on the noise
- TP = theoretical gain for positive weights only
- PW = optimum gain for positive weights only
- ZW = only positive weights larger than 0.05 included
- SB = standard beam gain in signal-to-noise ratio (SNR).

The above gain measures are given in dB relative SB. It should be remembered that it is important to mask properly 'dead' traces, as these otherwise would be given very large weights in view of their vanishing variances. On the other hand, spikes would be efficiently removed as such traces have large variances.

Real-time simulation of weighting scheme

To test the performance of the above weighting scheme, a limited number of noise samples for the new, prototype NORESS array (see Fig. VI.5.1) have been used in data analysis. Due to disk storage restrictions, only 13 channels have been used at any type: i) sensor 1, B & C rings and ii) sensor 1, C & D rings. The inner A ring sensors have been consistently deleted due to small ranges, as their sensor separations are relatively small vis-à-vis the upper frequency 'cut-off' at about 4 Hz. These sensors may well be used, but presently strong transmission (electronic) noise prevails above this frequency, i.e., the provisional 60 dB dynamic range recording system does not permit adequate noise sampling over a wide frequency range. Typical noise data used in analysis is displayed in Fig. VI.5.2; filter setting 1.0-3.0 Hz 3rd order Butterworth. The strong correlation between sensors in the innermost rings is rather obvious.

Noise suppression capabilities

The optimum gain function (negative weights) is visually displayed in Fig. VI.5.2, while more detailed results are presented in Table

VI.5.1 for a typical noise sample. Not unexpectedly, the largest noise suppression is obtained for low frequencies, that is, where the noise is fairly well correlated. This is also shown in the rightmost column in the table, where the number of times a sensor has contributed constructively to the weighted beamforming is indicated. This is tied to the 'positive' weighting scheme, but the weights contributing by less than 5% are zeroed while the remaining are set equal to 1. The total number of updating intervals was 44, so some sensors are truly abundant. The corresponding relative gain is listed in the ZW-column.

Some interesting features of Table VI.5.1 are as follows:

- the noise suppression is azimuth-dependent
- the weighting gain appears to be velocity-dependent, relatively best performances for surface wave phase velocities (4.6 km s^{-1}) and teleseismic P-velocities (12 km s^{-1})
- the weighting scheme can produce gains in excess of $\sqrt{N}$.

A similar experiment was performed by replacing the B-ring sensors with those of the D-ring (see Table VI.5.2). The outstanding feature here is the gain from weighted beamforming is very small, occasionally even negative. The reason for this is non-stationarity in the noise field when sensor separations become relatively large. The theoretical gain in this case is not too different from that of the previous and furthermore well above 1 dB. The latter data are not shown here.

The noise suppression is from Tables VI.5.1 & VI.5.2 obviously azimuth-dependent, and this feature has been further investigated. In Table VI.5.3 gain as a function of small azimuth intervals is displayed. Interestingly, for those azimuths where the gain (SB) is the smallest, the most contributing sensors (underlined in the rightmost colums) form triangles (squares) whose longest axis are roughly perpendicular to the azimuth in question (the OK notation in the AC column). This we interpret in terms of stronger correlation in the noise in the direction of propagation (radial) than in the transverse direction, which is indeed expected for scattering phenomena (Chernov, 1960). In other words, the correlation in the noise appears to depend on both sensor separation and their relative orientation. The latter feature, if prominent, would clearly have a bearing on the array configuration optimization schemes. For example, the approach of Mykkeltveit et al (1983) essentially amounts to a noise covariance matrix model with zeros or small negative values for all off-diagonal elements. We have not observed this during our analysis so far nor obtained a gain exceeding $\sqrt{N}$ for standard beamforming (SB). This is a bit puzzling as noise correlations as a function of sensor separations only have been observed to exhibit a small negative minimum of the order of 0.1 units. This discrepancy may reflect a certain non-stationarity in the noise field in short intervals as used here, although the above-mentioned noise covariance matrix model is likely to represent an oversimplification of the real noise field. Thus, if the off-diagonal elements cannot be ensured to be zero or negative, the consequence of this is that the center station seldom would contribute constructively towards noise suppression by beamforming. This is quite obvious from the results presented in Tables VI.5.1-3 for the lower frequency bands.

Discussion and conclusion

A preliminary evaluation of the provisonal regional array installed within the NORSAR array has been made. It is somewhat incomplete in the sense that the operational settings of the array up to now only permit analysis in the frequency band 1 to 4 Hz and thus excludes the most important frequency band for small local and regional events, say 1.5 to 3.5 m_b units, namely, 5 to 15 Hz. Anyway, the main results obtained are as follows:

- When there is a significant tie in the noise data, weighted beamforming would produce an additional noise suppression gain of the order of 1 to 3 dB, roughly equivalent to 10-30% operational improvement.
- The noise field must be rated non-stationary as significant changes in the noise covariance take place within 5 to 10 sec interval. In this respect claimed long term noise stationarity features

like negative correlation values in certain sensor separation ranges appear to be very modest, at least in comparison to relatively strong short term variations.

- A certain directability in the noise field is apparent for certain azimuth directions. In practice this gives reduced noise suppression capabilities or an equivalent higher false alarm rate on certain beams.
- Array configuration optimization; this is a problem if reasonable performance is desired over a relatively wide frequency range. Optimum processing schemes appear to be unavoidable here or the array must compromise on a duality in configuration.
- Finally, optimum weighting schemes are rather time-consuming in practice and can hardly be 'afforded' unless there is access to an array processor for handling the covariance matrix 'inversion' task. We are pursuing this problem now, the motivation being that this would be very cost effective per unit dB gained in noise suppression of small arrays.

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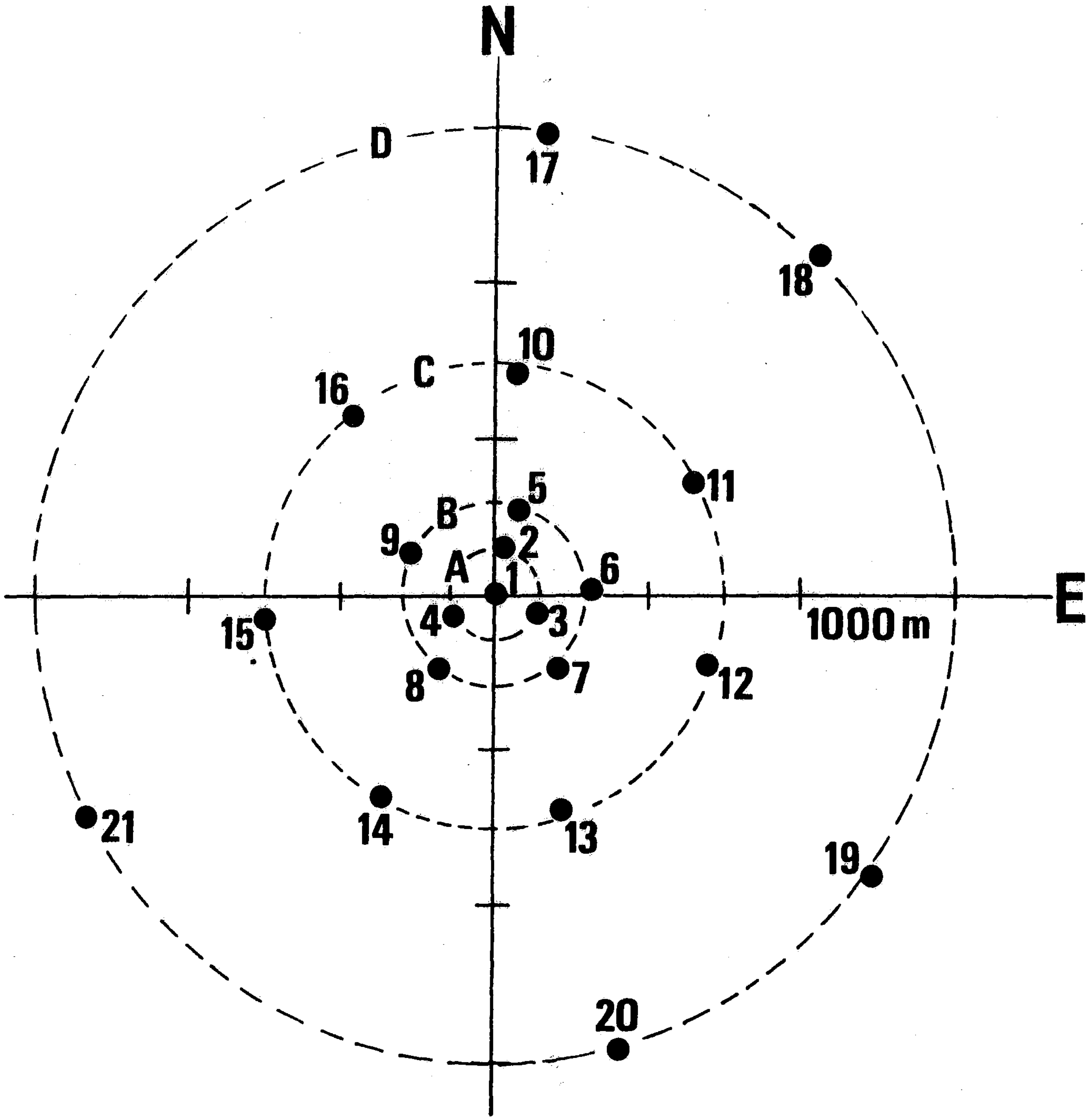

Fig. VI.5.1 The prototype NORESS array, data from which are used in this analysis.

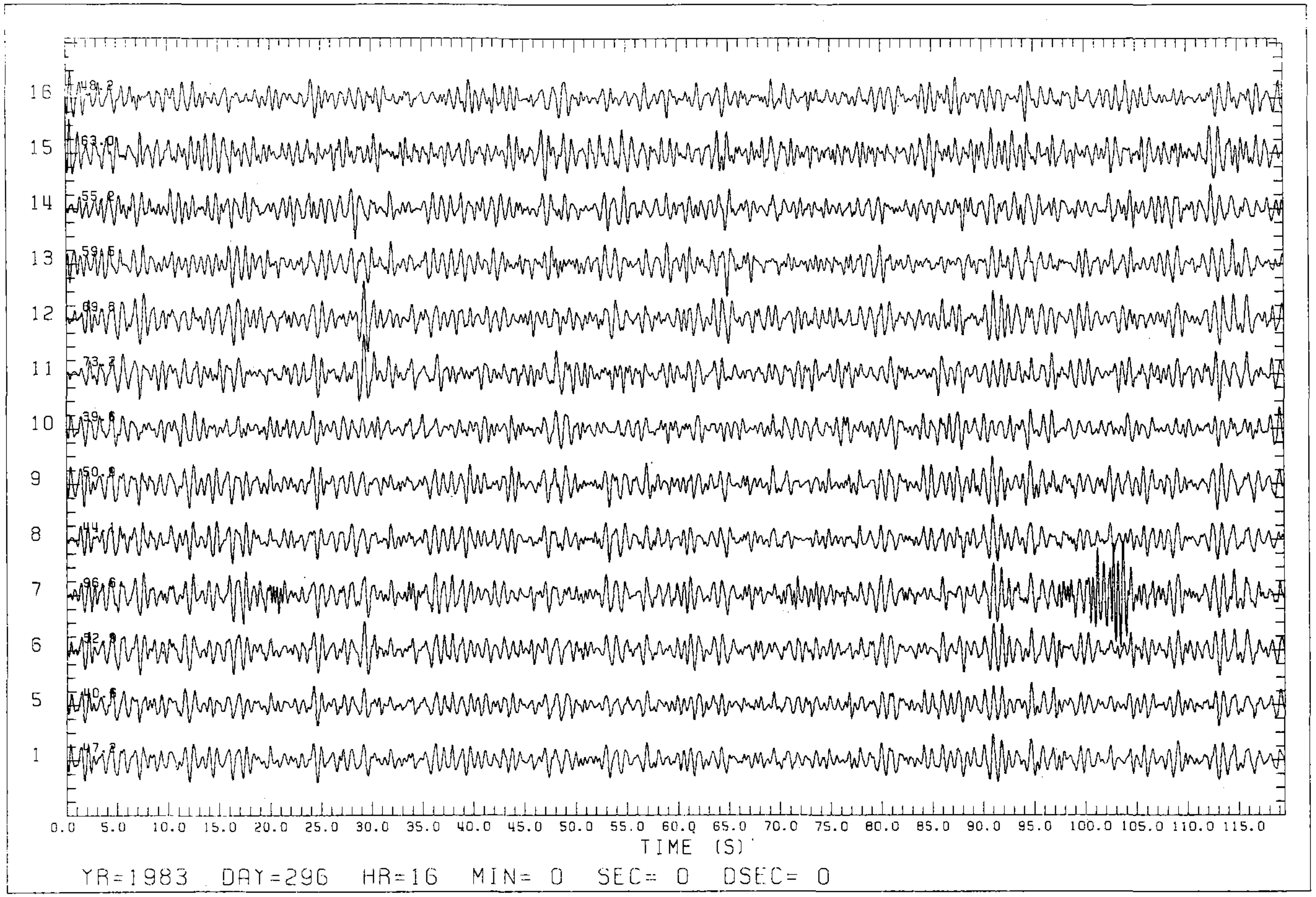

Fig. VI.5.2 Noise data used in analysis. Sensor numbering and rms-scaling on the vertical axis while the horizontal axis gives the time intervals (sampling rate 40 Hz). Filter 2 was used, i.e., the 1.0-3.0 Hz pass band.

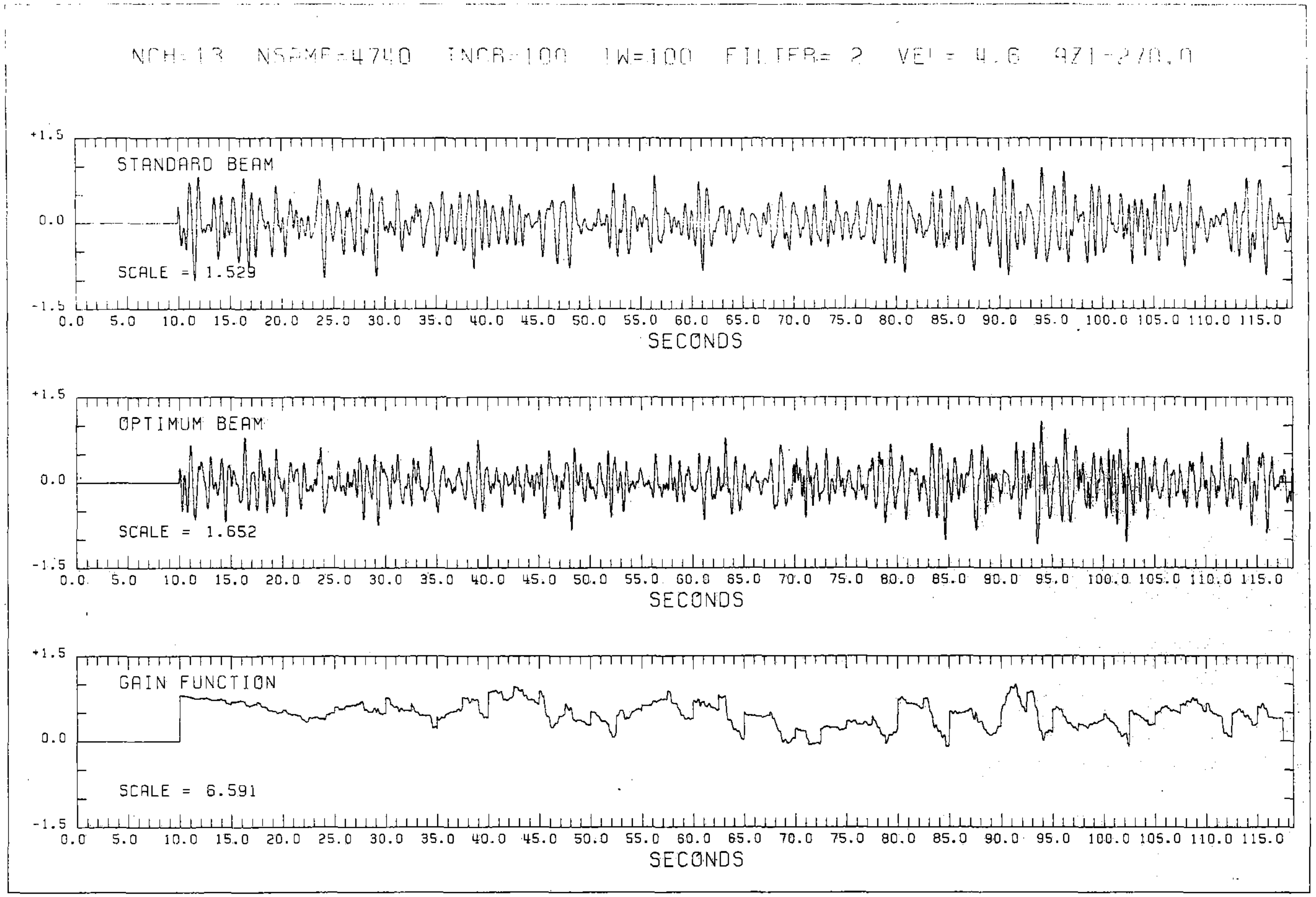

Fig. VI.5.3 Display of standard (BS) and optimum beam (ON) traces plus the corresponding gain function. The latter fluctuates rather rapidly in certain time intervals which we take to reflect a certain non-stationarity in the ambient noise.

Filter	Vel.	Az.	Gain				Al	B-ring					C-ring						
			SB	NW	PW	ZW	1	5	6	7	8	9	10	11	12	13	14	15	16
2	4.6	0	3.63	2.41	1.19	1.14	0	8	0	1	3	0	30	37	32	44	18	44	35
2	"	90	2.46	2.51	1.10	1.07	0	1	0	0	0	0	41	16	5	42	39	15	42
2	"	180	3.94	2.15	1.47	1.45	1	6	0	2	0	0	27	40	24	40	21	44	44
2	"	270	5.24	2.73	1.91	1.84	0	10	0	0	4	0	41	28	31	32	43	44	41
3	6.2	0	6.50	0.81	1.00	0.95	5	21	2	0	3	1	38	44	42	40	42	44	40
3	"	90	5.34	1.12	1.27	1.18	4	7	2	4	2	0	41	37	28	44	44	22	43
3	"	180	5.25	0.80	0.79	0.69	12	12	0	1	20	1	26	31	41	33	29	38	43
3	"	270	6.71	1.14	1.17	1.08	2	28	2	0	17	2	36	35	33	43	36	40	42
4	8.1	0	10.0	1.29	1.37	1.06	20	40	13	1	33	11	36	42	42	43	40	41	43
4	"	90	8.82	1.26	1.41	1.12	24	17	1	6	19	9	44	30	44	44	42	21	44
4	"	180	7.81	0.95	1.13	0.92	20	23	3	6	22	8	42	33	44	42	39	24	44
4	"	270	9.79	1.12	1.18	0.78	9	30	21	1	41	24	41	38	42	44	36	32	44
5	12.0	0	11.29	2.64	2.61	2.09	24	40	38	0	38	31	42	36	44	44	38	28	44
5	"	90	10.42	1.87	1.96	1.49	25	26	10	1	25	18	43	37	44	44	37	27	44
5	"	180	9.94	1.86	2.00	1.61	27	22	10	3	31	12	44	24	44	44	42	29	44
5	"	270	11.14	2.21	2.22	1.73	27	26	34	0	41	33	42	41	44	44	32	23	44

Table VI.5.1 Various measures of beamforming gains as a function of filter setting, velocity and azimuth. The Butterworth filters used are respectively 2: 1.0-3.0 Hz; 3: 1.5-3.5 Hz; 4: 2.0-4.0 Hz; and 5: 2.5-4.5 Hz. SB = gain (in dB) on the standard (unweighted) beam. The NW, PW and ZW columns give gain (in dB) relative to SB for beamforming based on both positive and negative weights, positive weights only and positive weights larger than 0.05, respectively. The columns associated with the Al, B- and C-ring sensors (see Fig. VI.5.1) indicate the number of times a sensor contributed to the ZW-beamforming scheme. For example, for filter 2 most of the Al & B-ring sensors hardly contribute to the noise suppression; their weights are less than 0.05, and besides the difference between PW and ZW gains is quite small. Total number of trials was 44.

Filter	Vel.	Az.	Gain				A1	C-ring							D-ring				
			SB	NW	PW	ZW	1	10	11	12	13	14	15	16	17	18	19	20	21
2	4.6	0	6.88	0.25	0.38	0.19	10	0	21	18	24	27	36	24	41	15	44	42	44
2	"	90	5.20	1.15	1.35	1.30	21	7	2	17	10	11	19	22	44	44	17	44	11
3	"	0	9.38	-0.36	-0.31	-0.88	9	13	17	29	17	30	29	30	44	44	42	40	44
3	"	90	8.26	0.48	0.57	0.26	8	35	34	15	36	22	3	26	44	38	26	44	43
4	"	0	10.73	-0.21	-0.21	-0.90	41	34	31	39	17	28	8	34	43	40	31	34	44
4	"	90	10.10	0.22	0.25	-0.13	39	44	31	13	44	34	0	40	44	30	19	42	44
5	"	0	10.58	0.18	0.19	-0.36	44	31	40	44	21	40	0	36	44	23	16	35	44
5	"	90	10.39	0.09	0.13	-0.33	44	44	43	36	44	32	0	43	44	8	10	40	29

Table VI.5.2 Caption similar to that of Table VI.5.1. Notice that a larger array configuration is used this time

Filter	Vel.	Az.	Gain				A1	B-ring					C-ring						
			SB	NW	TG	AC	1	5	6	7	8	9	10	11	12	13	14	15	16
2	4.6	0	3.63	2.73	3.54	-	0	5	0	0	1	0	32	41	35	44	19	44	39
"	"	30	2.92	2.00	3.34	ok	0	0	0	0	0	0	36	17	29	44	7	44	42
"	"	60	2.47	2.69	3.30	ok	0	0	0	0	0	0	39	21	8	44	21	33	44
"	"	90	2.46	2.77	3.50	ok	0	0	0	0	0	0	44	15	3	44	43	13	44
"	"	120	2.80	2.52	3.44	ok	0	0	0	0	0	0	44	25	3	40	44	36	18
"	"	150	3.28	2.33	3.36	-	0	0	0	0	0	0	44	36	15	29	42	42	15
"	"	180	3.94	2.29	3.45	ok	1	23	0	0	0	0	40	43	23	34	26	44	30
"	"	210	4.54	3.12	3.97	-	0	4	0	0	0	0	30	42	27	42	21	44	44
"	"	240	4.96	3.27	4.12	-	0	2	0	0	0	0	42	41	31	44	33	44	44
"	"	270	5.24	3.05	3.90	-	0	5	0	0	2	0	42	34	38	41	44	44	42
"	"	300	4.92	3.03	3.92	-	0	7	0	0	0	0	43	43	41	19	44	44	21
"	"	330	4.35	2.97	3.86	-	0	0	0	0	2	0	41	44	39	25	43	44	22

Table VI.5.3 Caption similar to that of Table VI.5.1. TG = theoretical gain (both negative and postive weights) while the array configuration (AC) column indicates for which azimuth direction there seems to be noise orthogonality, i.e., the noise correlation is more prominent in the radial (azimuth) than in the transverse direction. The most contributing sensors (underlined) form triangles or squares whose principal directions are roughly perpendicular to the beam azimuth.