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## VII.3 On the determination of radiated seismic energy and related source parameters

The determination of radiated seismic energy on the one hand, and of source size and static stress drop on the other, depends in principle on a representation of different parts of the source spectrum. In practice with band-limited data from a sparse network the required source parameterization is often the same. Spectral models parameterized by the source's central moments of degree zero and two are introduced as an approximation to the general representation of the amplitude spectrum in terms of the central moments of even degree. Phase spectra are not used, apart from polarity which is determined by a correlation method. These models are shown to simulate well the principal features of common circular and Haskell type models, including the corner frequency shift of P waves with respect to S waves, and the relation between rupture velocity and maximum seismic efficiency. Spectral bandwidth and the determination of radiated energy and apparent stress are contrasted to time domain pulse widths and the determination of source size and static stress drop in these models. In the last semiannual report we discussed the determination of spectral bandwidth and pulse width. Radiated energy and apparent stress are determined from a far-field representation involving the spectral bandwidth; source size and static stress drop are determined from a relation between the second central moments  $\lambda_x^2$ ,  $\lambda_v^2$  and an elliptical surface under uniform stress drop, i.e., we find the equivalent surface for uniform stress drop

 $S_{\sigma} = 5\pi \lambda_{x} \lambda_{y}$ (1)

and an estimate of the static stress drop is then

$$\Delta \sigma = 3M/(10\sqrt{5} \pi C \lambda_x \lambda_v^2)$$
<sup>(2)</sup>

where M is the scalar moment and the nondimensional constant C is a function of ellipticity  $\lambda_y/\lambda_x$ . Determining these parameters with a Gaussian spectral model gives results consistent with the limits imposed by

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cohesive cracks, i.e., the maximum seismic efficiency approaches one for rupture velocities near the limiting values (Rayleigh velocity for in-plane strain, S velocity for anti-plane strain). One of the results is shown in Fig. VII.3.1. An  $\omega$ -square model, although having more acceptable asymptotic properties, does not lead to these physically consistent results.

Since real data often require the number of source parameters to be limited, it is of practical interest to examine the consequences of a reduced number of parameters, in particular for circular models and point source approximations; in these cases results for radiated energy can be obtained in closed form. A comparison is given in Fig. VII.3.2, where the finite source and corresponding point source model have equivalent spectral bandwidth for P or S. Some general conclusions are as follows: "Equivalent" point source models severely underestimate the radiated energy from sources with strong directivity effects (e.g., Haskell type of models with rupture velocity near the limiting values), but they can give a useful approximation (within a factor of two) to the radiated energy from sources with relatively weak directivity effects (e.g., the class of circular models). It is possible to correct for the "point source bias" assuming a fixed directivity coefficient d (which is a function of source shape and rupture velocity). However, if d is fixed it follows that the seismic efficiency is also fixed, i.e., apparent stress will then be a fixed fraction of the static stress drop.

If the stress drop is constant and earthquake sources are geometrically and dynamically similar, then seismic energy scales linearly with moment. There is observational evidence both for and against the simple scaling relations. Source complexity will in general change these relations, unless the statistical irregularity of faulting follows a particular form of self-affinity. In a stochastic model characterized by some typical correlation distance of faulting, the scaling may be between linear and quadratic, and so the apparent stress would be

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between constant and linearly increasing with moment. Fig. VII.3.3 shows the results for a particular stochastic model specified in a recent paper (Doornbos, 1984). The results imply that a relatively large increase of radiated energy with moment would be accompanied by an underestimate of source size and an overestimate of stress drop. This bias is related to the misinterpretation of corner frequencies of complex faults, as noted by several authors (e.g., Madariaga, 1979). However, the determination of radiated energy may still be correct.

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## References

Doornbos, D.J. (1984): On the determination of radiated seismic energy and related source parameters. Bull. Seismol. Soc. Am., in press.

Madariaga, R. (1979): On the relation between seismic moment and stress drop in the presence of stress and strength heterogeneity. J. Geophys. Res., 84, 2243-2250.



Fig. VII.3.1

Maximum seismic efficiency  $\eta_{max}$ , and ratio of P to S radiated energy  $E_{\alpha}/E_{\beta}$  (multiplied with factor 10), versus directivity coefficient d. If d = 1, rupture velocity equals S velocity. Moment tensor approximation to Haskell unidirectional model, in combination with a Gaussian spectral model. The aspect ratio is 0.4.

: transversal slip; \_\_\_\_\_: longitudinal slip.



Fig. VII.3.2 Radiated seismic energy for point source approximations to Haskell unidirectional model (H1), Haskell bidirectional model (H2), circular model (C). Aspect ratio of Haskell models is 0.4. If rupture velocity equals S velocity, d = 1 in model H1, d = 2 in model H2 and  $d \approx 1.5$  in model C. Point source approximations are equivalent with respect to spectral bandwidth for S (\_\_\_\_\_) or P (\_\_\_\_). The curves give the fraction of radiated energy in relation to that of the true model.



Fig. VII.3.3 Ap

Apparent stress and stress drop versus moment of complex faults. The source is characterized by a stochastic part determining the spatial and temporal scale of faulting, and a deterministic part determining the spatial extent of the whole source; for details see Doornbos (1984). The reference moment and stress drop ( $M_0$ ,  $\eta\sigma_0$ ,  $\Delta\sigma_0$ ) are for a coherent subfault with scale lengths corresponding to those in the stochastic model.