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Technical Report No. 2/84

COMPUTER REPRESENTATION OF COMPLEX 3D GEOLOGICAL STRUCTURES USING A NEW 'SOLID MODELLING' TECHNIQUE

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H. Gjøystdal, J. E. Reinhardsen and K. Åstebøl

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ABSTRACT

During our recent work with 3D dynamic ray-tracing and velocity inversion problems, we have developed a new 3D model generation system, using a so-called 'solid modelling' technique. The term 'solid modelling' refers to the fact that the logical system governing the internal geometrical properties of the model describes the model as a combination of 'solids' or 'volumes' in 3D space. In each of these volumes the physical parameters (such as seismic velocity, density, etc.) vary continuously. Discontinuous changes occur only across the model interfaces separating the volumes.

The model is constructed by firstly forming a number of 'simple volumes' from the given interfaces and then combining these simple volumes into more complex volumes, which represent the physical volumes of the model. It is very easy to make changes to the model, as one may add volumes, subtract volumes, and perform more composite operations, all by simple use of Boolean expressions. Every time a model has been specified (or changed), the internal logic automatically carries out a check of physical consistency of the 3D model space (no overlapping volumes, no holes, etc.).

By including the use of various types of coordinate transformations, different kinds of complex structures can be handled, such as salt domes, vertical and near vertical faulting, etc.

INTRODUCTION

An essential part of a 3D seismic modelling system is the establishment of a computer representation of the geological structure to be modelled. The effectiveness and flexibility of the system is to a great extent dependent upon how the model is specified, how it can be changed, and how it can be automatically checked for physical consistency.

We have developed a model generation technique as part of a system for 3D dynamic ray-tracing. The ability to define complex models is especially important in modelling by ray-tracing, as this method is very useful in areas of complex geology. Ray-tracing allows modelling of the reflections from a particular part of a structure without computation of the entire wave field; thus it is possible to simulate and investigate closely and quickly the seismic response from a single, interesting feature inside a complex structure.

The geological model used in ray-tracing is a set of 'solids' or 'blocks' separated by 'interfaces'. The blocks represent physical volumes in which the physical parameters such as P- and S-wave velocities and density are assumed to vary continuously. The dynamic ray-tracing method requires that each interface can be described as a continuous mathematical surface having continuous first and second partial derivatives (Cerveny et al, 1977), however, the boundary between two adjacent blocks may be given as a combination of several 'smooth' interfaces. The model must be 'consistent', that is, every point in the model must be inside one and only one block.

Very few works in geophysical literature have been devoted to the problem of constructing and representing 3D 'volume models' with a relatively high degree of complexity. In 3D, we are faced with a number of difficulties that are not present in the more commonly used 2D case, such as

- interfaces may intersect along general spatial curves
- considerably more complex mathematical representations are needed to describe interfaces and physical variables
- difficulties in specifying the various continuous volumes (blocks) in an unambiguous way
- problems with proper model display and consistency check.

In computer-aided design and manufacturing (CAD/CAM), similar model generation systems are used (Requicha and Voelcker, 1982). However, in CAD/CAM the models are usually defined as combinations of analytic surfaces, which simplify the model definition significantly. This restriction is unacceptable in geophysical applications. Boundaries between volumes may have almost any shape. The model representation is further restricted by the strong continuity requirements inherent in the dynamic ray-tracing method.

The present work introduces and demonstrates a new 3D model generation system designed for geophysical modelling in general and for dynamic ray-tracing in particular. Like recently developed model representation systems for CAD/CAM, this will be called a <u>solid modelling technique</u>, due to the fact that the logical entities in the model are the 'solids' or 'volumes'. Most earlier CAD/CAM systems, however, cannot be classified as solid modelling systems, as they are purely 'surface oriented', that is, designed for displaying combinations of surface elements in space without relation to the volumes between them. Such a 'surface modelling system' has great limitations relative to a 'solid modelling system'. For example, there exists no logic inherent in the system that is able to determine if an arbitrary point in space is inside a certain volume bounded by the surfaces, even if the surfaces actually define such unambiguous closed volumes.

In order to illustrate the use of the models, a number of dynamic ray-tracing examples have been included.

3D MODEL CONSTRUCTION

A solid model representation consists of two parts:

- a representation of the interfaces in the model
- a logical structure defining the volumes in terms of the interfaces.

Fig. 1 gives an example of a very simple 3D model, consisting of

- a number of interfaces, and
- a number of volumes between the interfaces.

The 3D model shown in Fig. 1 is simple for the following reason:
All interfaces are defined throughout the whole model volume, and none of these interfaces intersect. This results in a very simple logical

structure for the model: A point (x,y,z) lies in volume no. i if and only if (x,y,z) lies below interface i and above interface i+1, and this is valid for all i=1,...,N-1.

Fig. 2 shows a somewhat more complex model, having interfaces that are not defined throughout the entire model volume, and in addition, some interfaces end towards others (intersect). In this case it is far more complicated to establish a 'logical structure' for the model, i.e., a procedure which is able to determine which volume an arbitrary point (x,y,z) in the model belongs to. As opposed to the former case, it is no longer sufficient to refer to the volume between two interfaces. For example, the points A and B in Fig. 2 belong to the same volume V_2 but are still located at opposite sides of both interfaces 3 and 4. It is thus not sufficient to use a simple above/below-relationship in order to determine which block a certain point belongs to.

In the following we shall introduce a system for defining a 'logical structure' of the model even in such general cases, and by a couple of examples try to illustrate the idea of how to construct specific volumes in space and combine them to the physical volumes wanted. The more rigorous mathematics underlying the system has been omitted.

Fig. 3 illustrates the basic brick-stone of the model. The model is defined as the volume inside the rectangular box (V_M), and xyz is the basic model coordinate system. Generally, each model interface is represented in a local coordinate system $u_i v_i w_i$ (i is the interface index) as a function

$$w_i = f_i(u_i, v_i) \qquad (u_i v_i) \in A \qquad (1)$$

where A is the area of definition in the $u_i v_i$-plane. (In Fig. 3 B_1 and B_2 corresponds to the projection of the area A onto the model boundaries.)

The local coordinate system may in principle be any coordinate system. In most cases the interfaces are represented in local cartesian coordinate systems rotated and translated relative to the basic model system. However, in special cases other types of coordinate systems are used, e.g., cylindrical coordinates are useful for representation of folds, and spherical coordinates may be used to described dome-shaped structures, as shown in Fig. 4. The function f_i may be any type of continuous, bivariate function with continuous first and second partial derivatives. In most cases the interfaces are originally given by sets of sample points in space, and consequently some type of mathematical function must be fitted to the data. We usually employ bicubic splines; they have the necessary continuity properties and will in most cases fit the original interface samples reasonably well (Gjøystdal et al, 1984).

The interface i defines two volumes (in Fig. 3 denoted by W_i^1 and W_i^{-1}), which are the volumes on the positive and negative side of the interface, respectively (positive side is in the direction of w_i). Volumes like

W_i^1 and W_i^{-1} constitute the elementary volume elements of the model. By combining such 'interface volumes' in various ways, the full 3D model can be constructed.

In order to construct more complex model elements, we shall introduce two classes of volumes: simple blocks (SB) and complex blocks (CB). A simple block is simply the intersection between a number of specified 'interface volumes' of type W_i^1 or W^{-1} defined above. That is, an SB consists of all points in space which are inside all these interface volumes simultaneously. For example, we may define an SB as

$$V_{SB} = W_1^1 \cap W_2^{-1} \cap W_3^1 \tag{2}$$

which means all points in space being on the positive side of interface no. 1, on the negative side of interface no. 2, and on the positive side of interface no. 3, simultaneously. A simple illustration of this example is given in Fig. 5. The three interfaces are given in separate cartesian coordinate systems (Fig. 5a), and the simple block (V_{SB}) results in the closed volume inside the hatched part of the interfaces (Fig. 5b). If the local coordinate systems and areas of definition for the interfaces are coincident, the interpretation of an SB is very simple: The simple block is the volume below some given interfaces and above some other ones, as shown schematically in Fig. 6. An important property of the SB is that it is not necessary to define explicitly the intersection curves between the interfaces. The simple block definition ensures that non-physical extensions of the mathematical representations of the interfaces do not affect the block.

Most volumes with simple geometry can be represented by simple blocks. There is one important exception: when non-physical, mathematical extensions of the interfaces intrude the volume, it cannot be described as a simple block. The reason for this is that a point inside the volume may be located both above and below an interface bounding the volume, such that simple above/below relationships cannot be used. This was the case for the model in Fig. 2, and another example is shown in Fig. 7. In such cases, the volume must be defined as a complex block, which is a logical composition of other blocks, simple or complex ones, combined by means of Boolean algebra. Four different operators are available: union, intersection, subtraction and complement, and any combination of these can be used. Fig. 8 shows a simple example of CB definition. As demonstrated in this example, it is sometimes necessary to make simple blocks that do not represent physical volumes, but are needed in the design of complex block. Fig. 9 gives a number of additional examples, illustrating the use of the various Boolean operators. We observe that the generation of CBs is usually very simple, although the CBs themselves may be rather complex.

We now return to the model of Fig. 2, and show how this model can be constructed by use of the SB/CB logic (see Fig. 10). We assume that all the interfaces are given in the basic model system, that is, $u_i v_i w_i$ = xyz for all i. Firstly we generate five 'simple blocks'

(SB_1-SB_5) by combining positive/negative sides of the interfaces. Note that interfaces 3 and 4 have been mathematically extended (dashed lines). SB_1 and SB_4 represent the physical volumes V_1 and V_4, respectively. From the simple blocks SB_2, SB_3 and SB_5, complex blocks representing the model volumes V_2 and V_3 are formed by Boolean algebra. As soon as the various 'complex blocks' have been defined, different 'volume functions' representing physical parameters can be assigned to each block.

Fig. 11 demonstrates the ability of the model representation system in the construction of complex 3D models. The model shown has been divided in a number of 'slices' to visualize its true 3D nature. Steep and vertical fault planes are easily represented in properly rotated cartesian coordinate systems. Notice how the SB-CB logic allows definition of volumes with very complex boundaries.

PROPERTIES OF THE SOLID MODELLING SYSTEM

As mentioned, this model generation system is a solid modelling system. The volume logic has some very advantageous properties which are difficult to obtain for a purely surface-oriented model representation.

The most important property is that it is easy to determine whether an arbitrary point in the model is inside or outside a given volume. This is the fundamental feature of solid modelling systems, as it is the basic information needed if the model shall be treated as a set of solids. A valuable consequence for ray-tracing applications is that sources and receivers can be put anywhere in the model. Another important consequence is that model consistency can be automatically checked. Check points are selected on a 3D grid in the model with a predefined spacing, and it is checked that each point is inside one and only one block. The grid spacing can in principle be chosen arbitrarily small, in order to fit the model complexity.

It is very easy to modify the model geometry. As soon as a number of SBs has been established, it is easy to combine them into CBs and to combine the SBs and CBs into even more complex CBs and so on. Each time the model has been changed by altering the Boolean expressions, it can be rapidly and automatically checked for physical consistency by the computer. Two volumes may be easily combined to one by taking the union, and a new block can be put into another one by assigning the new block to the model and subtracting it from the original block. Thus, the user can very easily manipulate with the model blocks in a way that is not possible without a full 'volume logic'.

An example of model modification is illustrated in Fig. 12. The diapiric structure in Fig. 12a will be put into the model in Fig. 12b. The diapir is represented by a separate complex block, and first we assign this block to the model. Then we subtract by Boolean algebra this block from the other blocks in the model, and a new, consistent model including the diapir is obtained (Fig. 12c). This procedure is very simple, as it is not even necessary to know in detail what blocks the diapir intrudes. If the diapir is subtracted from a block which in fact is completely outside the diapir, it follows from the properties of Boolean set subtraction that this block remains unchanged.

Interfaces may also be modified. If the modifications only change the shapes or locations of some interfaces, without altering the logical relationships between the volumes and the interfaces, no new logical build-up is needed. Translation and rotation of interfaces, leaving their shapes unchanged, are particularly simple, as it is done by redefinition of the local coordinate systems for the interfaces. The interface functions f_i are unchanged.

EXAMPLES OF DYNAMIC RAY-TRACING

We now turn to the application of models generated by the new solid modelling technique by including a couple of examples from 3D dynamic ray-tracing.

Fig. 13 shows two synthetic zero offset seismic sections computed for lines on the surface of the model in Fig. 12c and parallel with the shown cross section. Fig. 13a shows a plot of selected ray paths for a line with equally spaced source-receiver pairs across the top of the diapir. The diapir is rotationally symmetric, and the other interfaces are two-dimensional and invariant in the direction normal to the shown cross section. Fig. 13b shows the corresponding synthetic seismic section, containing only in-line reflections due to the model symmetry.

The large hyperbola-like reflection in the middle of the section comes from the top of the diapir. Otherwise some typical features of seismic sections across diapirs are easily recognized: the terminations of the reflections around the diapir indicate its size, and there is a characteristic velocity pull-up of the reflector below the diapir. The section in Fig. 13c is computed for a line outside the diapir, and side reflections are clearly recognized.

A simple non-zero offset VSP-example is shown in Fig. 14. The shot point is buried a little below the surface, and the receivers are equally spaced down the well (Fig. 14a). Some selected ray-paths for direct rays, primary reflections and surface multiples are shown in the cross section Fig 14b. Fig. 14c is the synthetic VSP section.

CONCLUSION

Complex, 3D geological models can be effectively generated by means of the presented solid modelling technique. The model representation consists of two parts: mathematical representations of the interfaces and a logic defining the volumes in terms of the interfaces. Each interface is represented by a bivariate function in a local coordinate system. By proper choice of coordinate system (rotated, cartesian, cylindrical, spherical), most types of interface geometries occurring in seismic work can be described. To define volumes of different complexity, two classes of blocks are employed: simple and complex blocks. A volume into which the mathematical extensions of the interfaces do not intrude, is defined very simply as a simple block. A volume of more composite geometry must be given as a complex block, i.e., as a combination of other blocks by means of Boolean algebra. In practice, any realistic model in seismic modelling can be represented by this solid modelling system.

The models can be easily modified. Interfaces may be moderately changed without altering the logical model build-up. On the other hand, boundaries between volumes can be removed and new volumes put into the model by simple Boolean operations. The model consistency can be automatically checked after each model modification.

ACKNOWLEDGEMENTS

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FIGURE CAPTIONS

Fig. 1 Illustration of the volume/interface relationship for a 'simple' 3D model. S_1 - S_5 are model interfaces, and V_1 - V_4 are model volumes. Volume V_i consists of all points below S_i and above S_{i+1}.

Fig. 2 Illustration of a 'complex' 3D model which does not have a simple volume/interface relationship like the 'simple' 3D model shown in Fig. 1. S_1 - S_7 are model interfaces and V_1 - V_4 denote volumes.

Fig. 3 Illustration of the volumes $W_i{}^1$ and $W_i{}^{-1}$ associated with an interface function f_i. The cube represents the model volume. Areas B_1 and B_2 represent the projection of the area of definition A_i of f_i onto the model boundaries. The interface function f_i divides the 'tube' between B_1 and B_2 into $W_i{}^1$ and $W_i{}^{-1}$ which are the volumes on the positive and negative side of f_i, respectively.

Fig. 4 Illustration of a dome-shaped interface function $r = f_i(\theta,\phi)$ given in a spherical system θ,ϕ,r. θ,ϕ,r are defined relative to a system x'y'z' translated and rotated relative to the basic system xyz. In this case the local coordinates (u_i,v_i,w_i) are equal to (θ,ϕ,r).

Fig. 5 Example of 'simple block' definition.

a) Three interfaces given in separate coordinate systems (u_1,v_1,w_1), (u_2,v_2,w_2), and (u_3,v_3,w_3), respectively. V_M is the total model volume.

b) The hatched area shows the simple block defined by

$$W_1^{1} \cap W_2^{-1} \cap W_3^{1}$$

W_1^{1}, W_2^{-1} and W_3^{1} are shown in a).

Fig. 6 xz-cross section through a simple block (hatched area) where all the bounding interfaces are defined in the same coordinate system and have coincident areas of definition. A is the common area of definition. The logical structure of the simple block V is:

$$W_1^{1} \cap W_2^{-1} \cap W_3^{-1}$$

In this case this simply means that volume consists of all points below S_1 and above S_2 and S_3.

Fig. 7 Cross section through a volume (hatched area) which cannot be described as a simple block. The bounding interfaces S_1 and S_2 intrude the volume. Some parts of the volume V are in W_1^{1} while other parts are in W_1^{-1}, and the same applies to W_2^{1} and W_2^{-1}, hence the interfaces S_1 and S_2 cannot be used in a definition of the volume as a simple block.

Fig. 8 Cross section through a volume defined as a complex block.

Firstly, two simple blocks are defined:

$$SB_1 = W_1^{\,1} \cap W_3^{\,-1}$$

and

$$SB_2 = W_2^{\,1} \cap W_3^{\,-1}$$

The complex block representing the physical volume is the union of these two simple blocks:

$$V = SB_1 \cup SB_2$$

Fig. 9 Illustration of 'complex block' (CB) build-up.

a) and b) show five 'simple blocks' (SB) and c)-f) show examples of how these simple blocks may be combined to complex blocks by simple Boolean expressions.

Fig. 10 Construction of the model in Fig. 2. The logical structure defining the volumes is:

SB1	$W_1^{1} \cap W_2^{-1}$	(below 1 and above 2)
SB2	$W_2^{1} \cap W_3^{-1}$	(below 2 and above 3)
SB3	$W_3^{1} \cap W_6^{-1}$	(below 3 and above 6)
SB4	$W_6^{1} \cap W_7^{-1}$	(below 6 and above 7)
SB5	$W_4^{1} \cap W_5^{-1}$	(below 4 and above 5)

V_1 = SB1

V_2 = SB2 $\cup$ SB5

V_3 = SB3 - SB5

V_4 = SB4

Fig. 11 Series of cross sections through a comlex 3D model generated by means of the model representation above.

Fig. 12 Illustration of model modification. The diapiric structure in a) is assigned to the model in b) and subtracted from the other blocks in this model. The result is a new, consistent model including the diapir shown in c).

Fig. 13 Examples of dynamic ray-tracing.

a) shows the model and a number of selected normal incidence ray paths. b) and c) show synthetic zero offset seismic sections computed for two parallel lines across the surface of the model. Only primarly reflections are included.

Fig. 14 Non-zero offset VSP example of dynamic ray-tracing. The model with shot point and well is shown in a), and some selected ray paths are shown in the cross section b). c) shows a synthetic seismic section, including the direct wave, primary reflections and multiples.

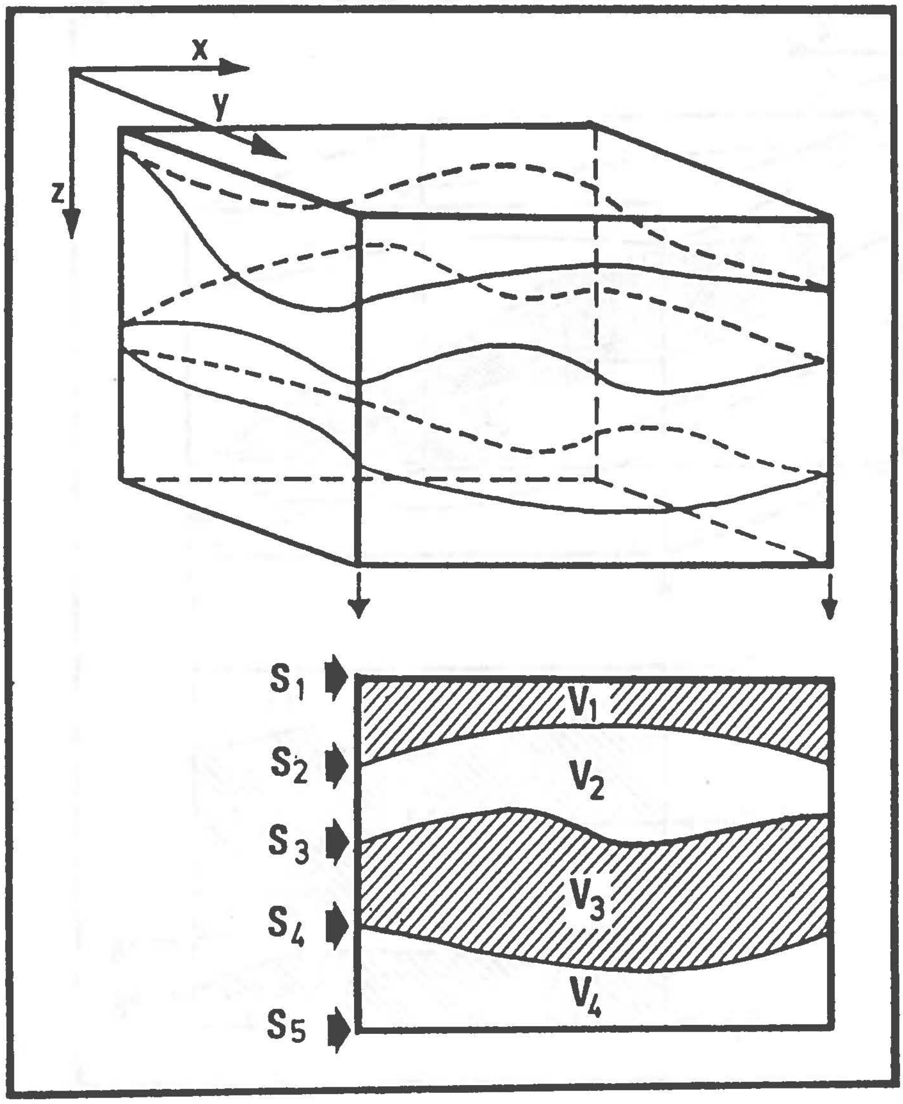


Fig. 1

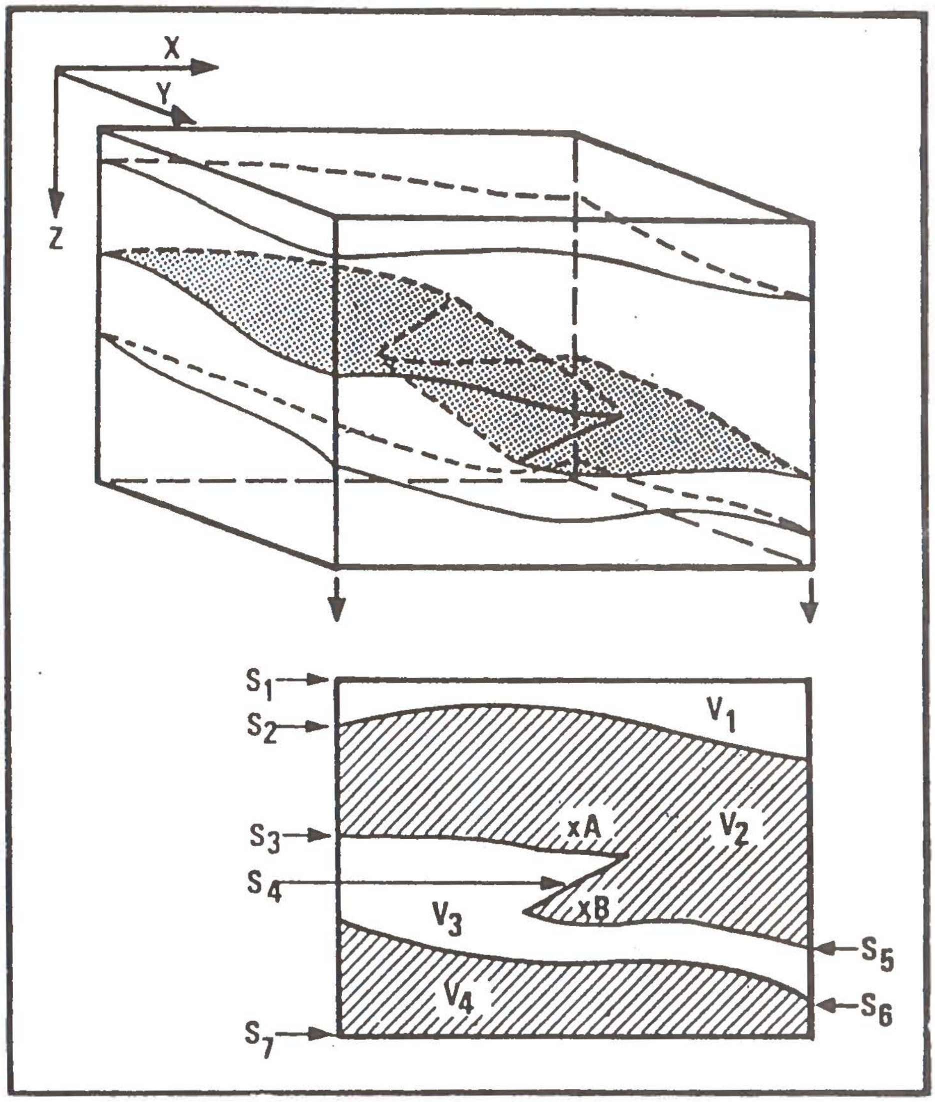


Fig. 2

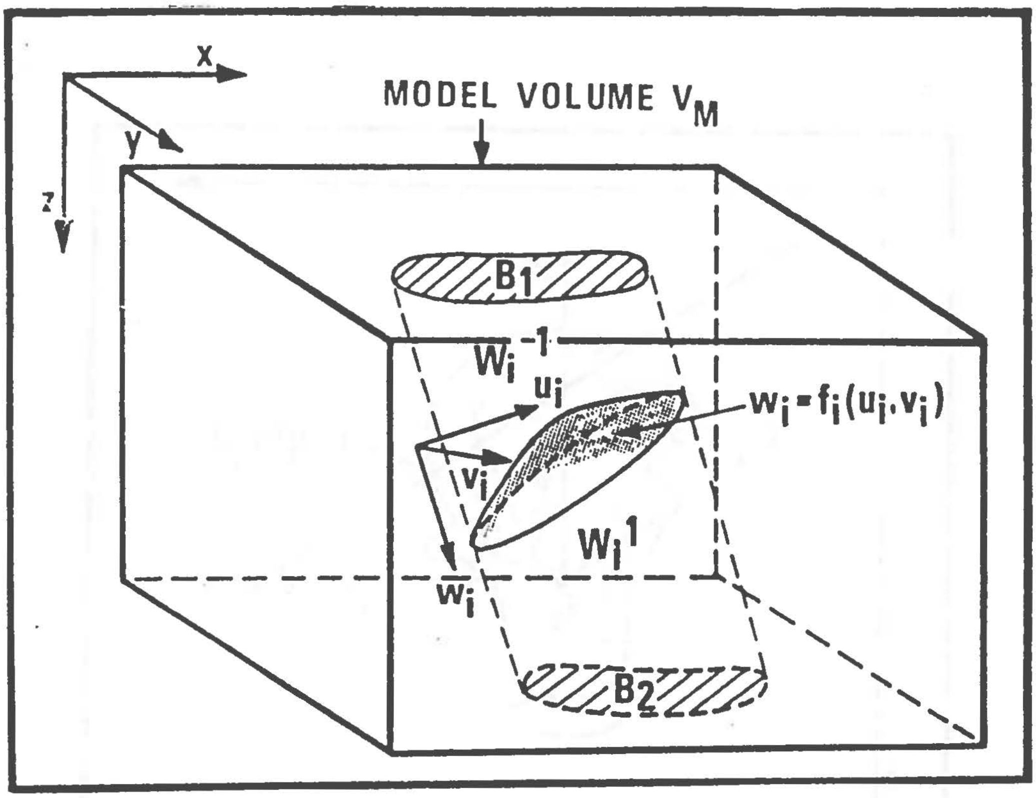


Fig. 3

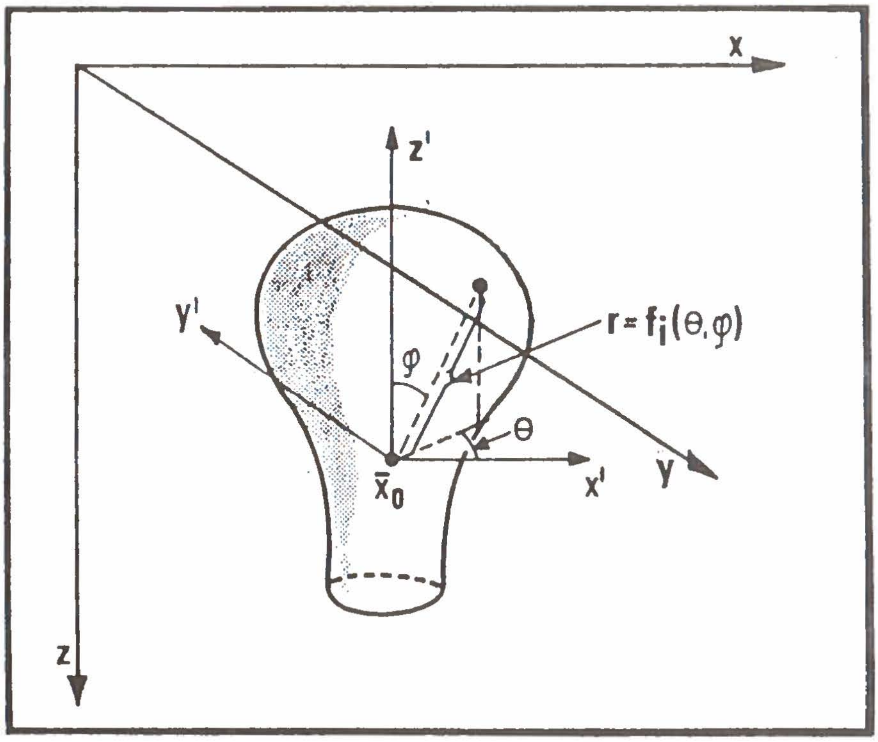
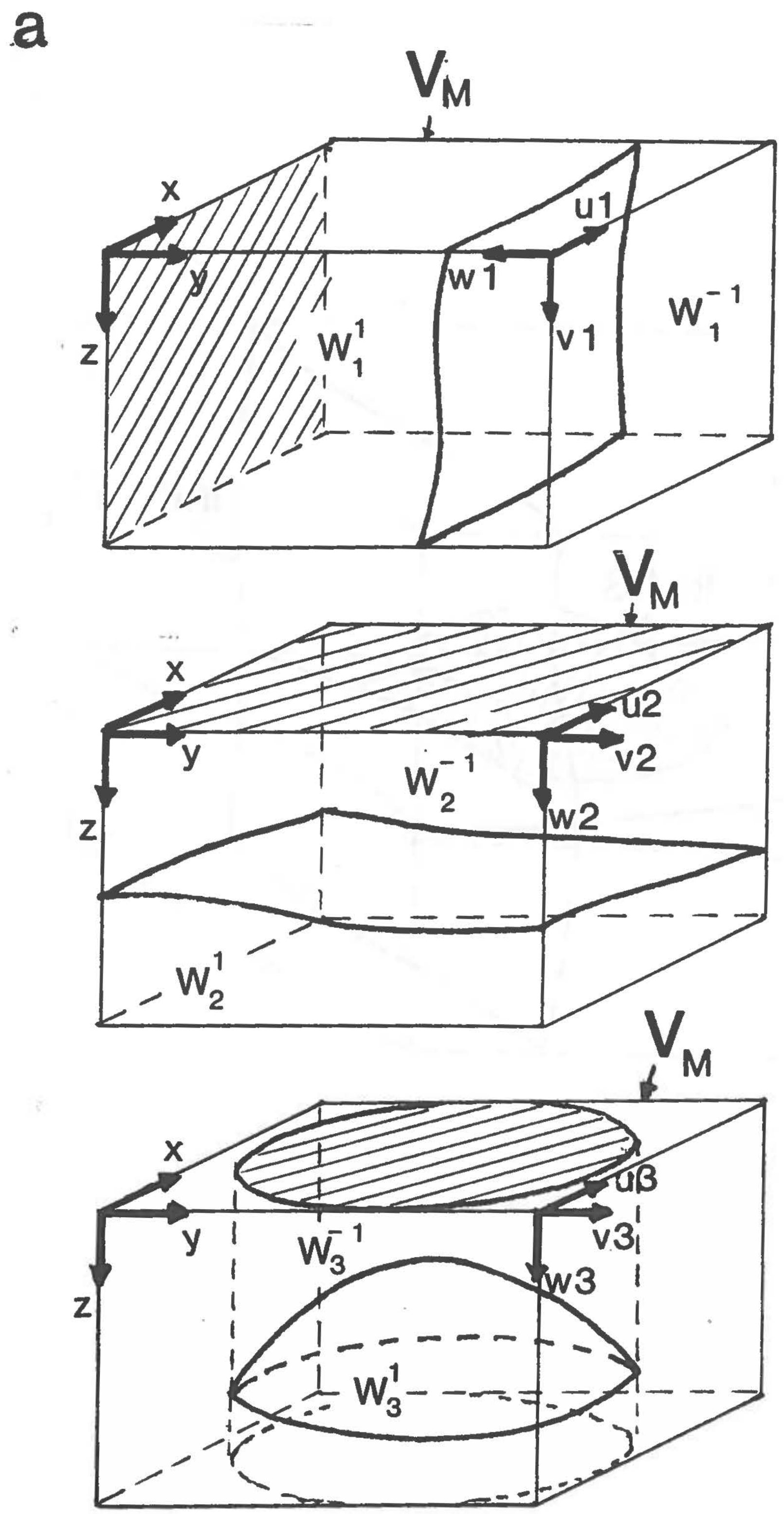


Fig. 4



Interface 1:

Nearly vertical interface represented in the system $u_1v_1w_1$:

$$w_1 = f_1(u_1,v_1)$$

Interface 2:

Nearly horizontal interface represented in the system $u_2v_2w_2$:

$$w_2 = f_2(u_2,v_2)$$

Interface 3:

Bowl-shaped interface represented in the system $u_3v_3w_3$:

$$w_3 = f_3(u_3,v_3)$$

Hatched areas: Area of definition for each interface function.

Fig. 5a)

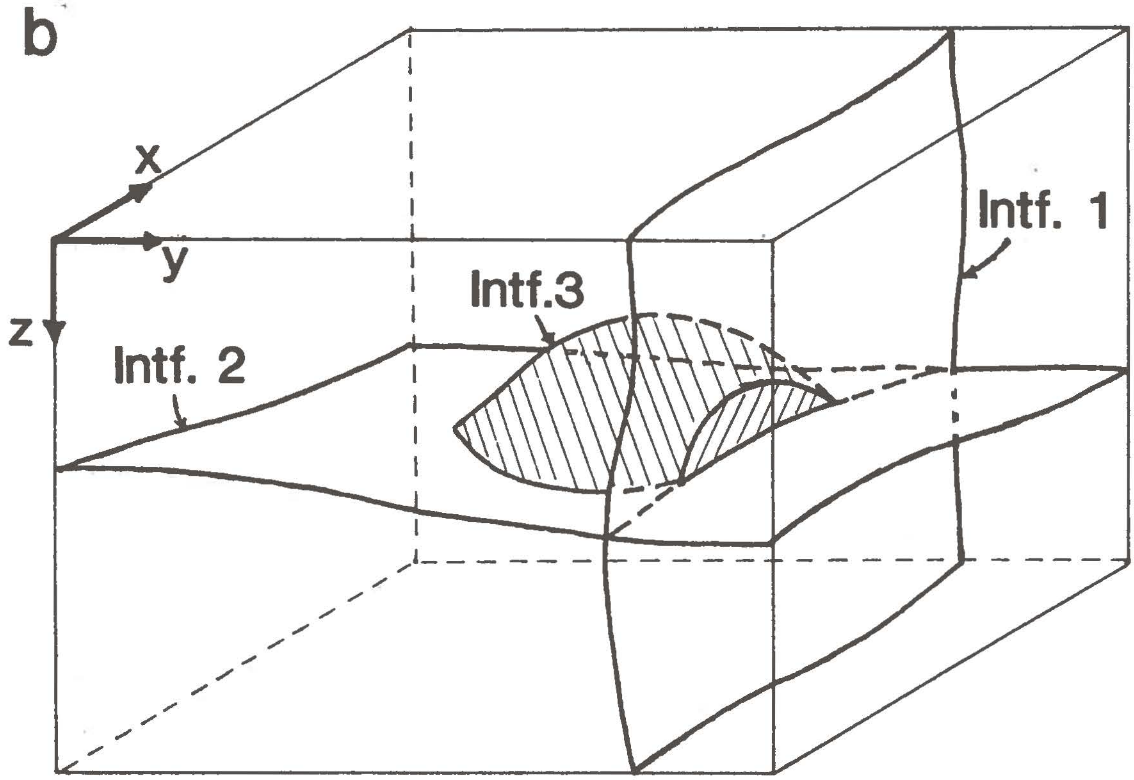


Fig. 5b)

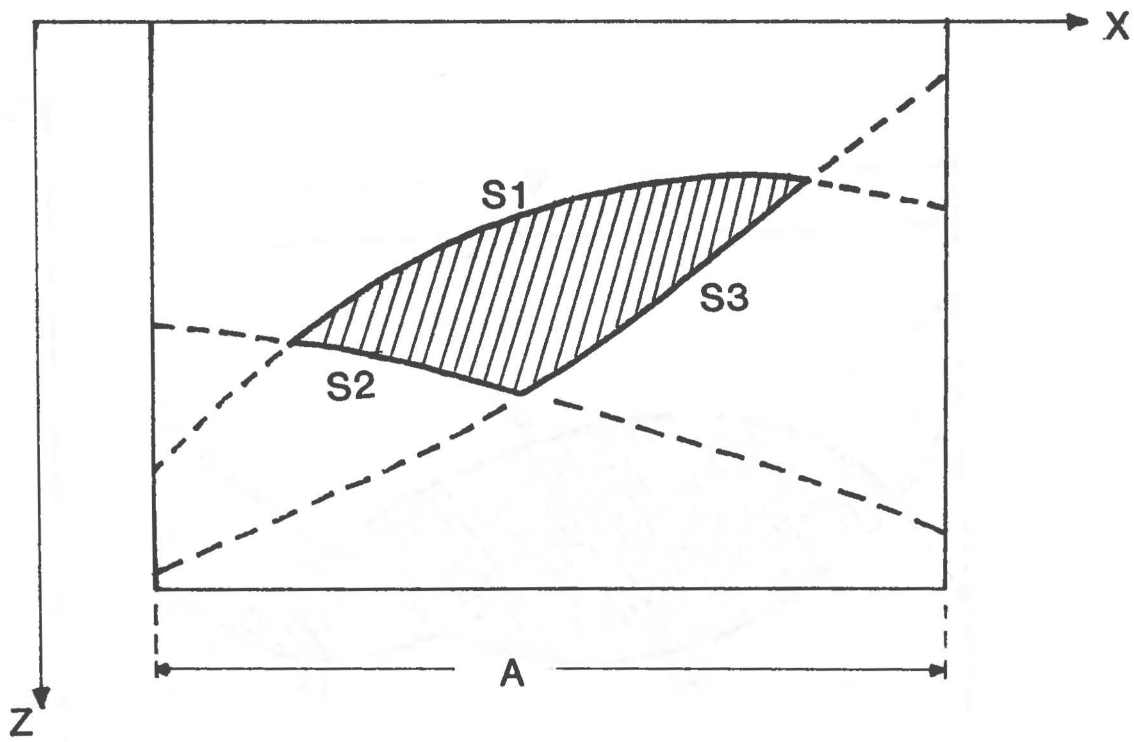


Fig. 6

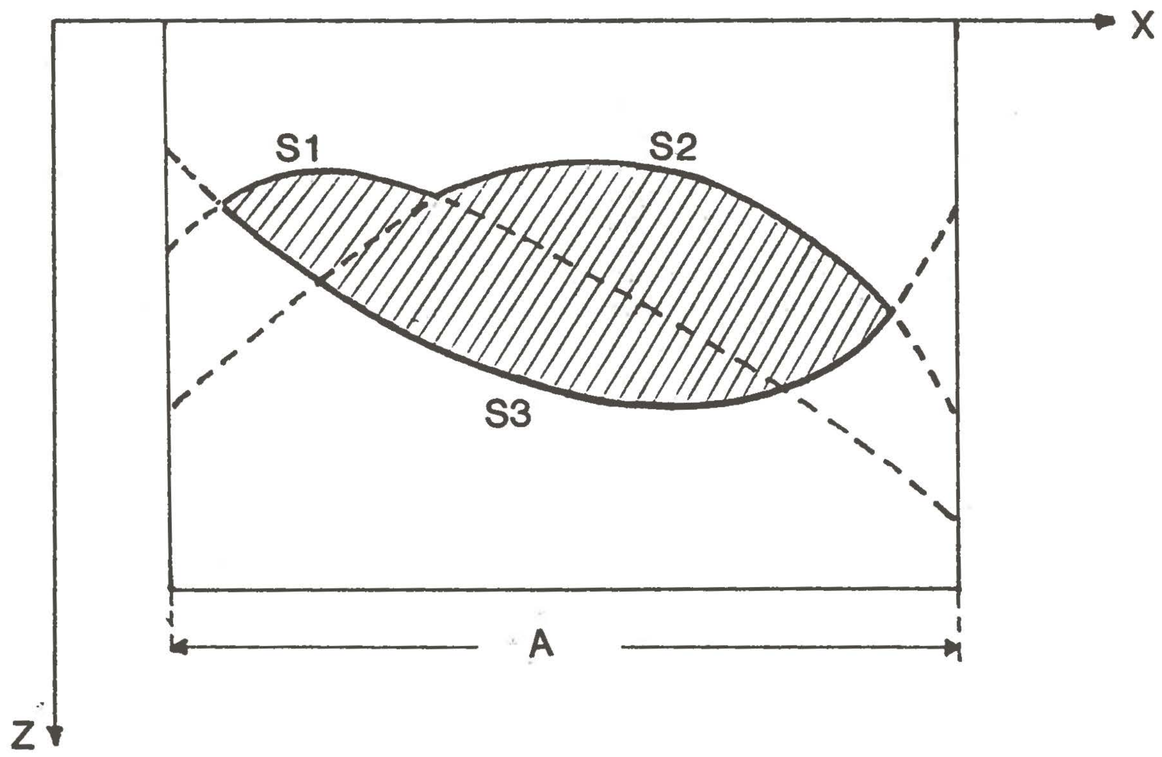


Fig. 7

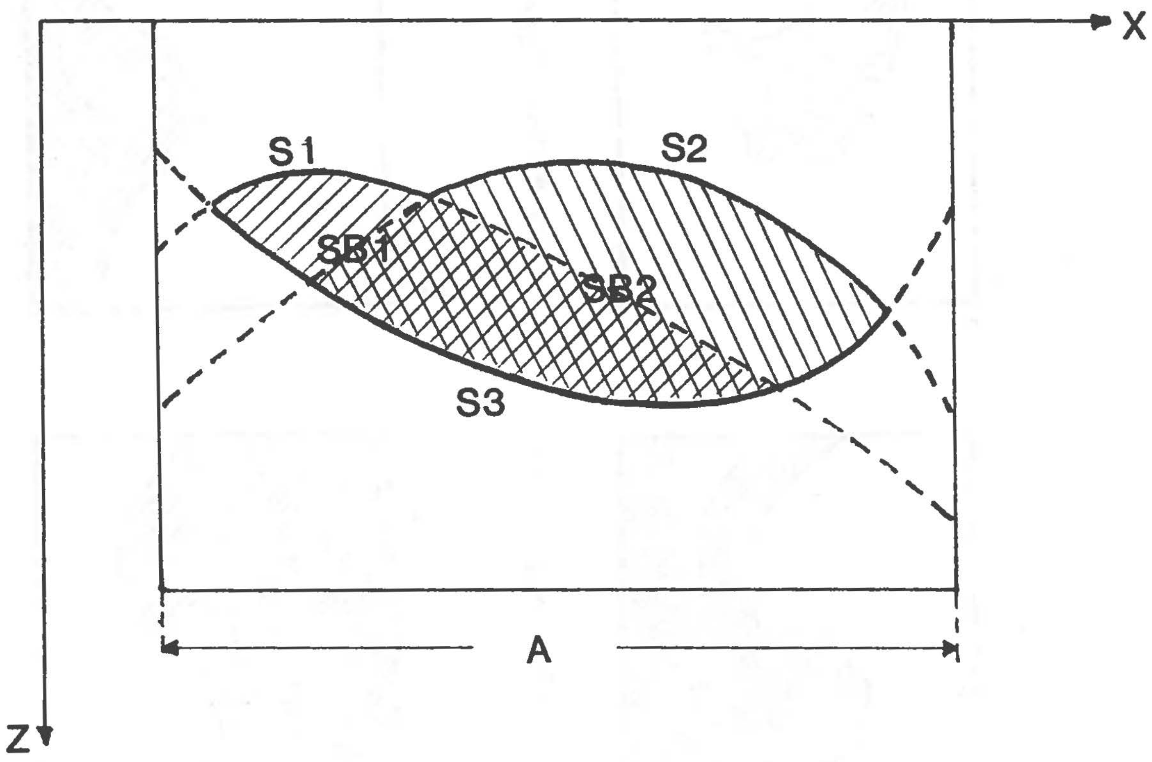
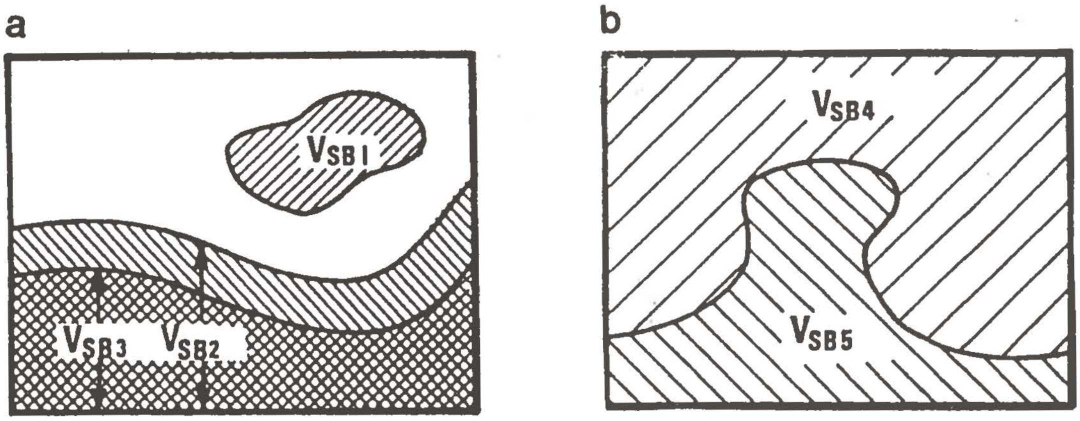
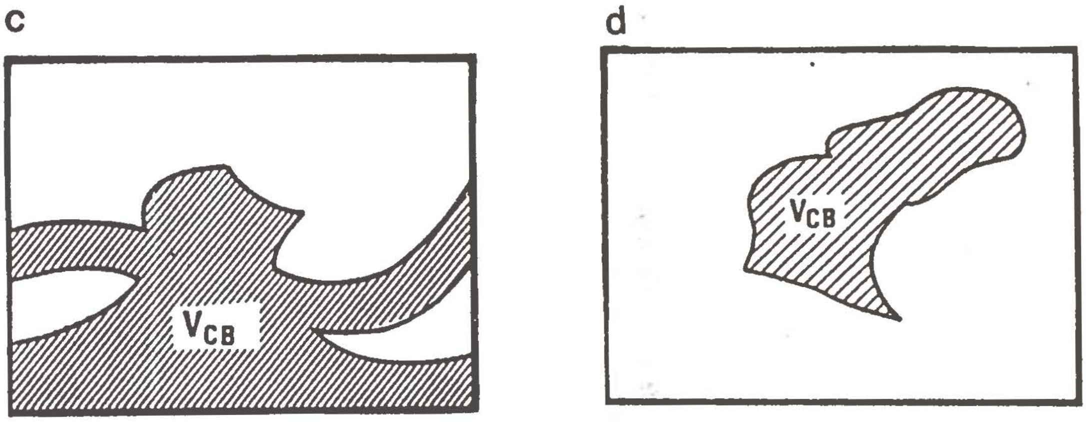


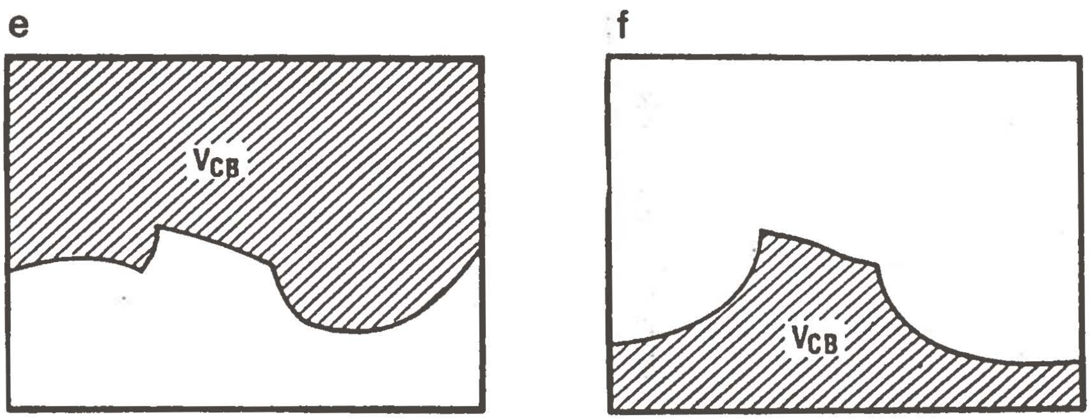
Fig. 8





$V_{CB} = (V_{SB5} \cup (V_{SB2} - V_{SB3})) - V_{SB1}$

$V_{CB} = (V_{SB5} - V_{SB3}) \cup V_{SB1}$



$V_{CB} = \complement V_{SB3} - [(V_{SB2} - V_{SB3}) \cap V_{SB5}]$

$V_{CB} = V_{SB5} - \complement V_{SB2}$

$= V_{SB5} \cap V_{SB2}$

Fig. 9

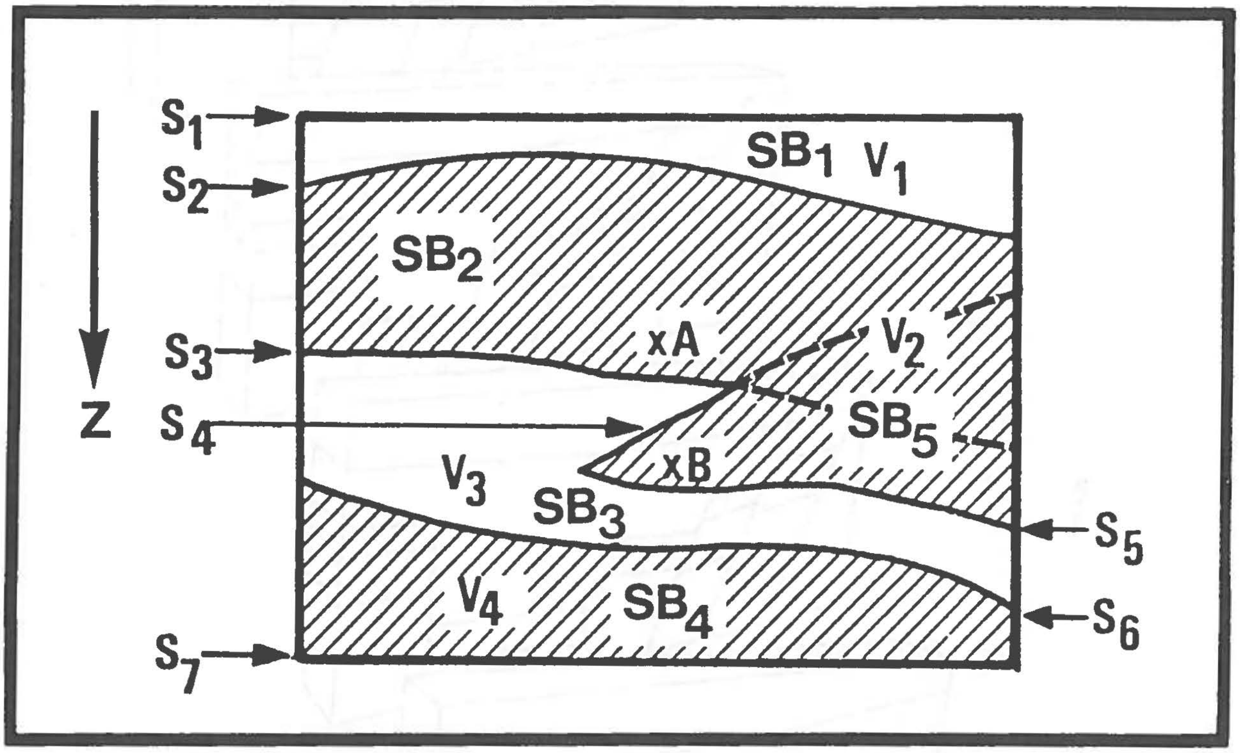


Fig. 10

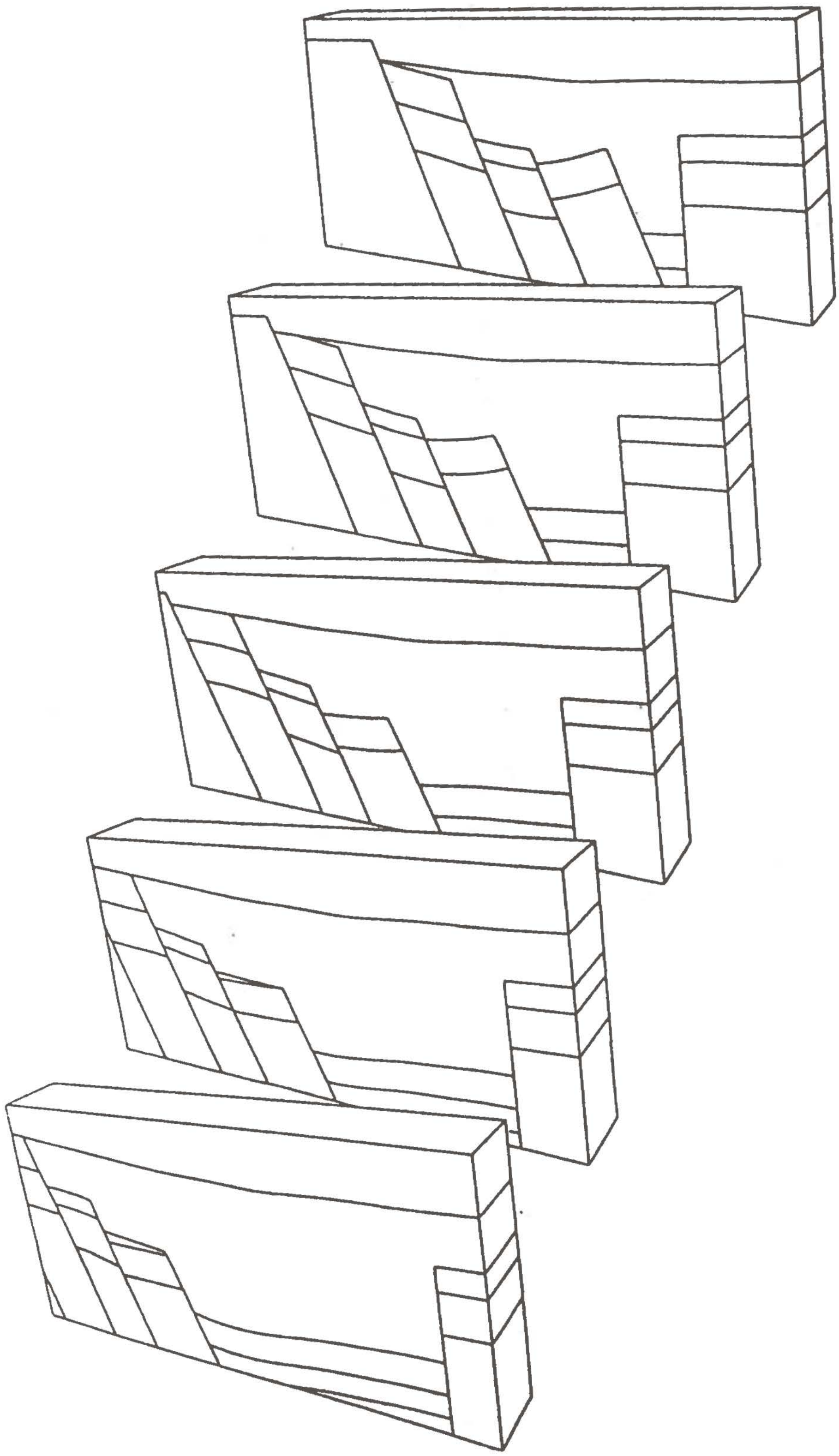


Fig. 11

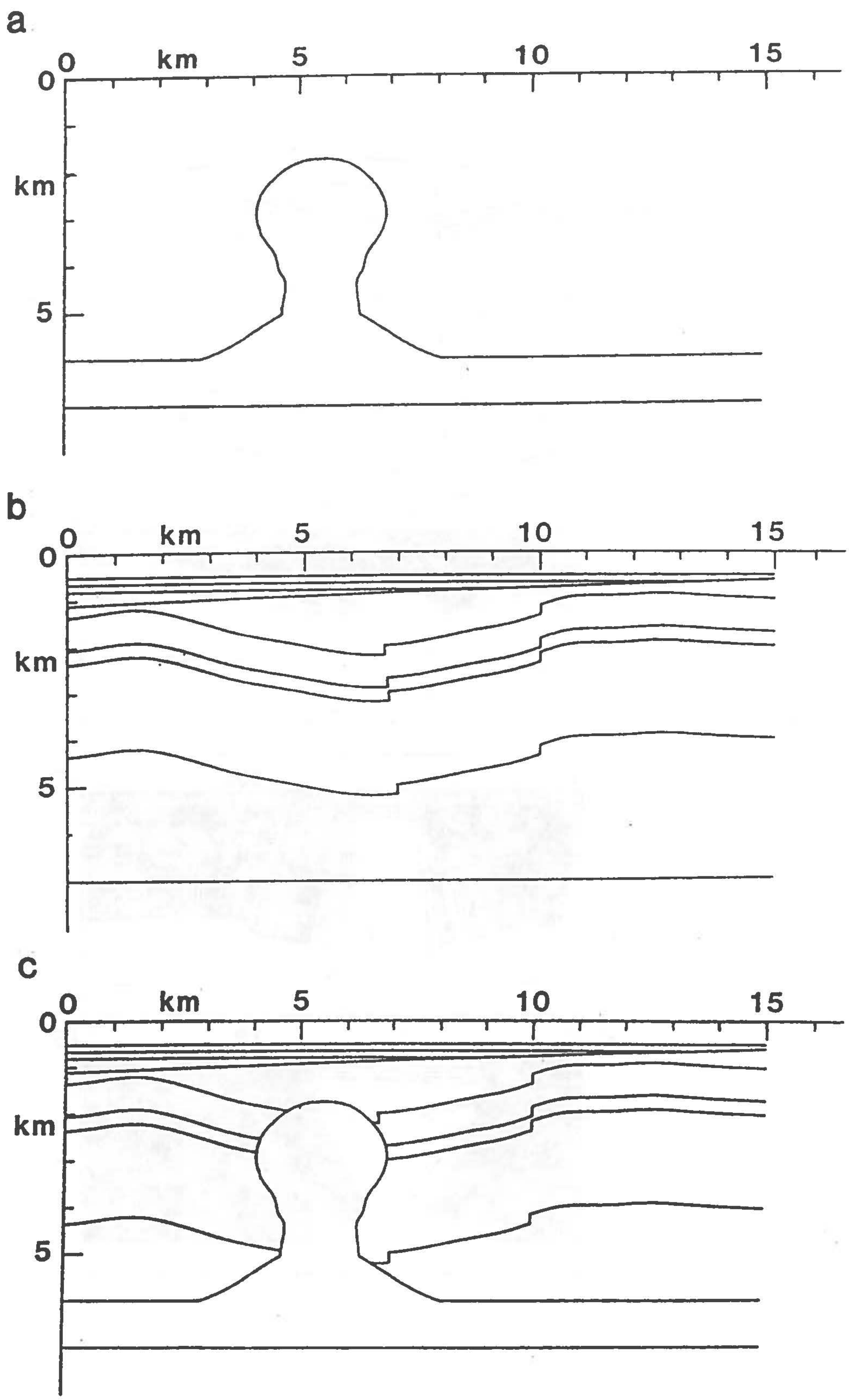


Fig. 12

a)

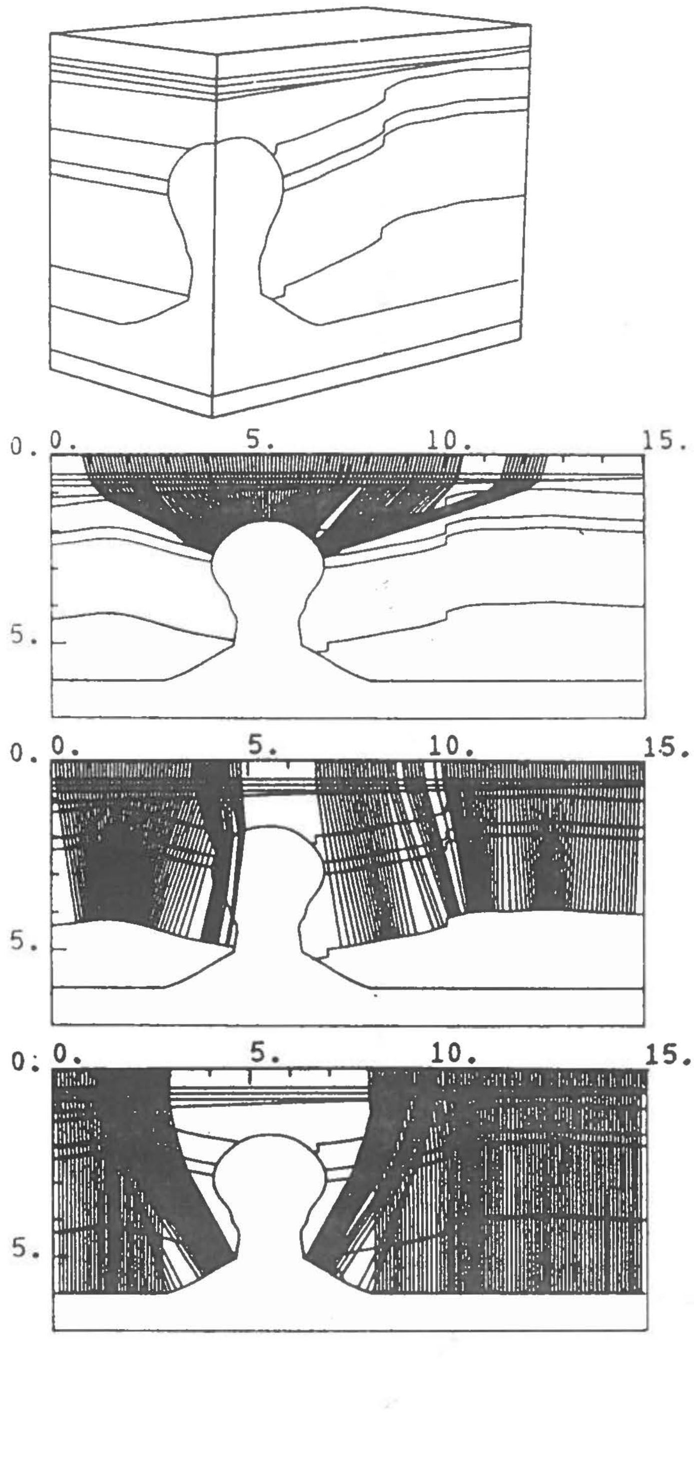


Fig. 13a)

b

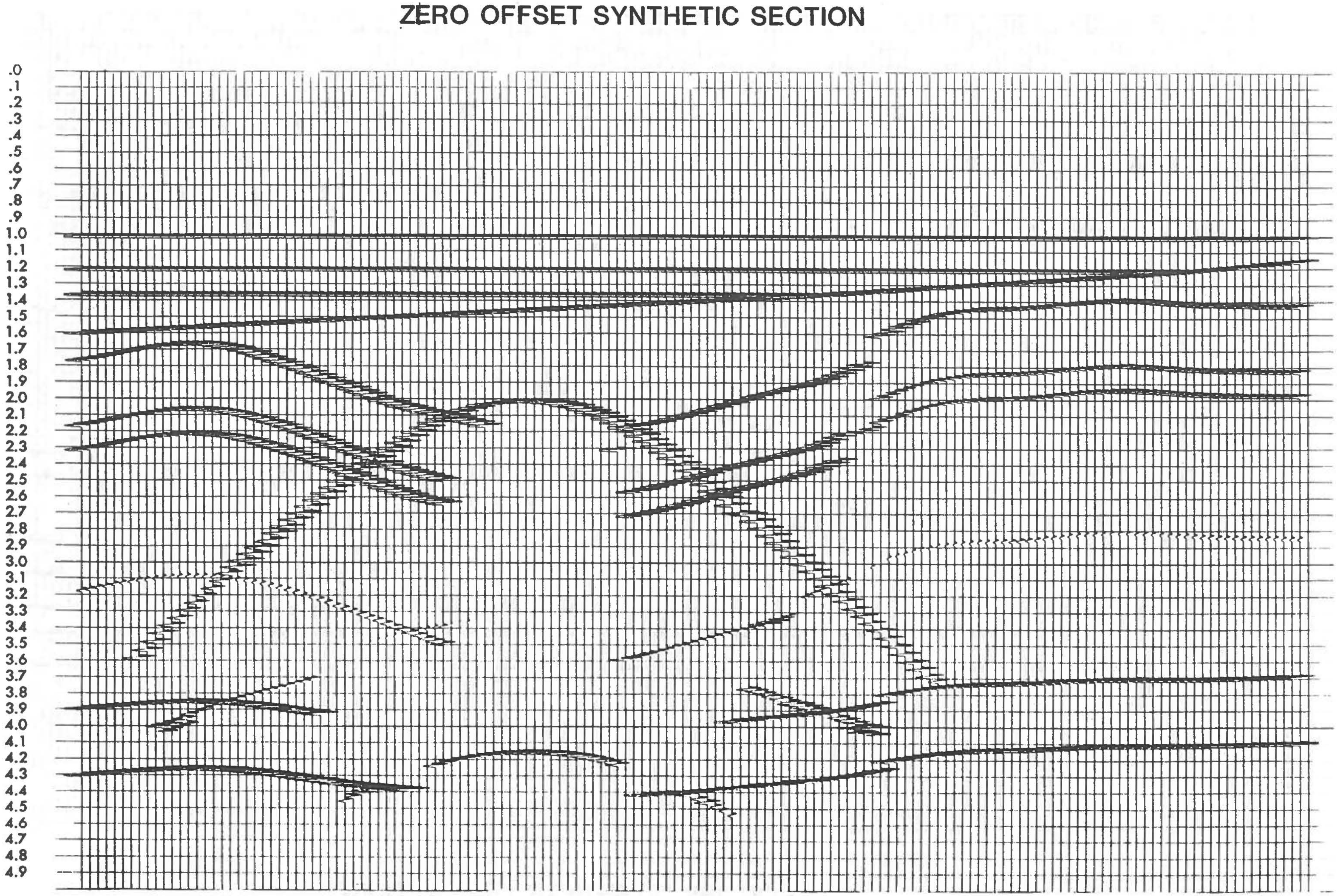

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Fig. 13b)

C

ZERO OFFSET SYNTHETIC SECTION

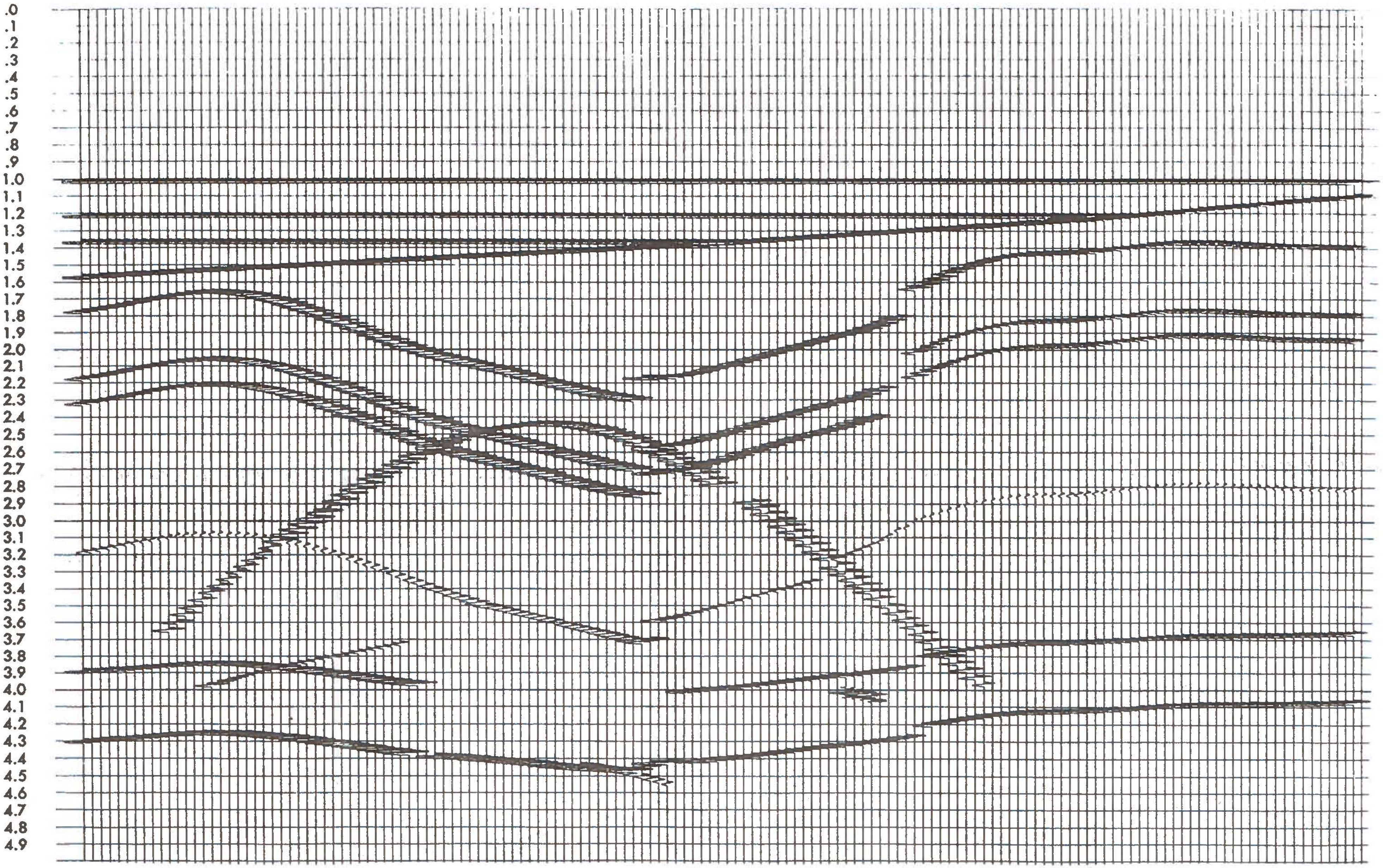

Fig. 13c)

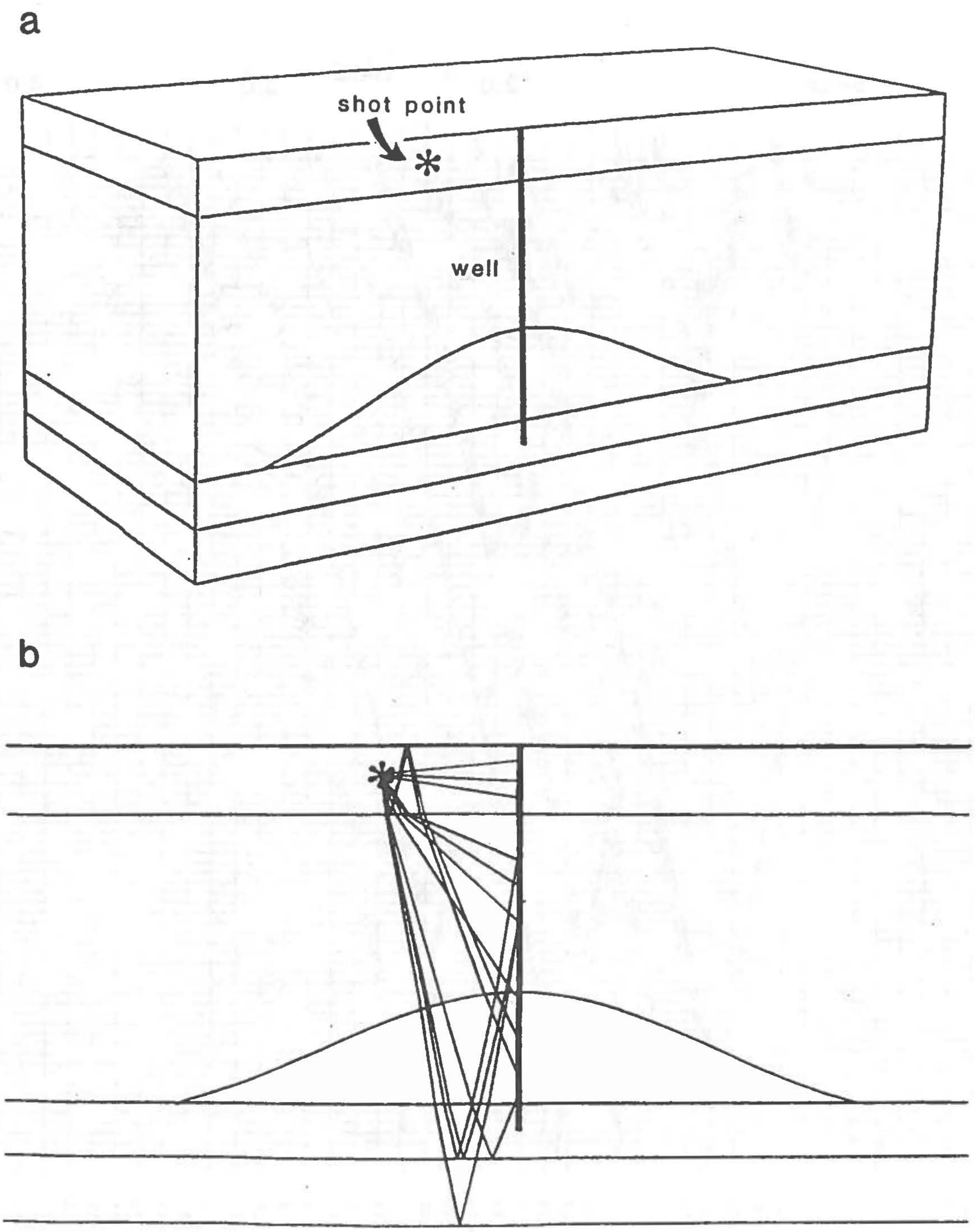

Fig. 14a)+b)

VSP - SYNTHETIC SEISMIC SECTION

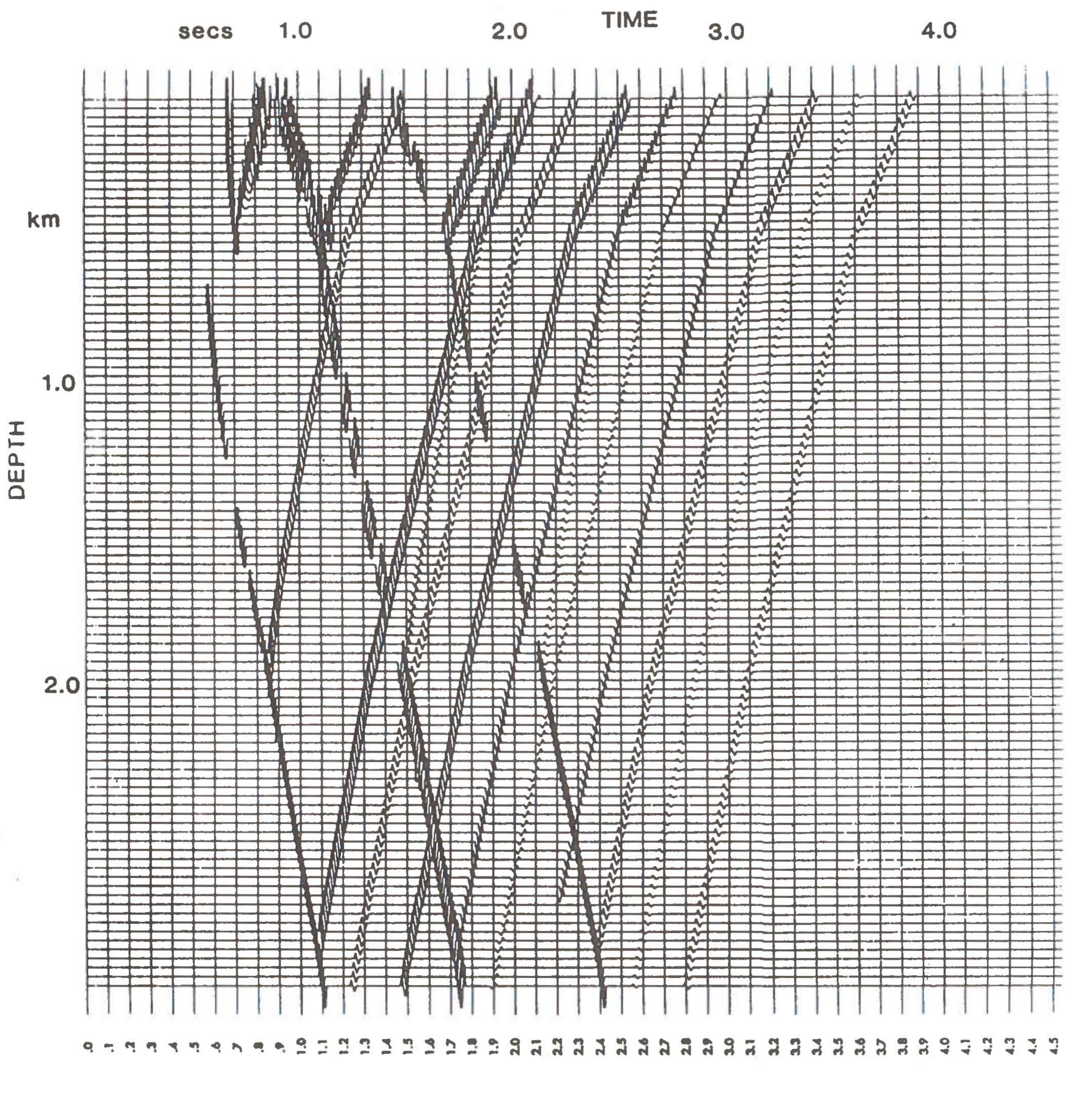

Fig. 14c)