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VII.3 Preliminary report on a method of dynamic ray tracing

The project, started in October 1983, is still in progress. The objective is numerical implementation of the methods of dynamic ray tracing (of seismic waves) developed by the author. These methods encompass (1) propagation through caustic regions; (2) edge diffraction; (3) the effects of anisotropy and prestress.

Algorithms and FORTRAN programs for kinematic ray tracing and dynamic ray tracing have been worked out by A. Hanyga. Preliminary testing against simple analytical models (SATM2, SATM3) has been carried out by A. Hanyga on IBM at NORSAR, partly in collaboration with O.A. Sandvin. Testing against realistic models has been done by Dr. Jan Pajchel at the Seismological Observatory, University of Bergen, on VAX.

The programs trace rays and signals in arbitrary user-supplied models. The models are represented by model data files and by a library of subprograms called by the ray tracing programs. We now describe various aspects of the methods applied.

Dynamic ray tracing through caustic regions

It has been shown by Hanyga (1984, 1984a, 1984b, 1984c) that dynamic ray tracing can be generalized to cover the case of WKBJ signals propagating through caustic regions. The generalized DRT consists of integrating a system of ordinary differential equations and is based on a natural extension of the WKBJ method originated by Maslov (Maslov 1972) in quantum mechanics (see Chapman & Drummond, 1982, for an alternative application of the Maslov-WKBJ method in the theory of seismic wave propagation).

The wavefield at a receiver situated at some distance from the caustics is evaluated by tracing the signals along the rays connecting the receiver to the source. The field at a point close to a caustic is given by a diffraction integral whose integrand can be evaluated by tracing along a family of rays.

At a distance from the caustics DRT is carried out by integrating the usual system of ordinary differential equations along a ray. The unknown functions include the parametric equation of the ray (in terms of the travel time parameter), slowness, wavefront curvature matrix and geometric ray spreading referred to the ray-centered coordinates as defined by Hanyga (1982). In the case of S waves the polarization angle is also included. This task is performed by a special subprogram of the DRT program.

Whenever the absolute value of the geometric ray spreading J turns out to be smaller than a positive number ε a canonical transformation in the phase space of position and slowness vectors is performed and a transformed version of the DRT equations is integrated. This task is performed by another DRT subprogram. As soon as the geometric ray spreading exceeds $\varepsilon > 0$ the original DRT is resumed.

The transformation of the phase of the signal implicitly contains the phase shifts at the caustics. This is a nontrivial advantage of the method in the case of anisotropic media where the phase shift cannot be guessed by a simple argument (see Chapman & Drummond, 1982, in the case of isotropy).

The transformations referred to above are purely algebraic. The DRT system for the caustic regions differs from the usual one merely by the number of equations (12 instead of 10). In the case of a ray which does not exactly touch a caustic the transformation improves the accuracy.

Boundary conditions are taken from Hanyga (1984a) and initial conditions (at point sources) from Hanyga (1984d). DRT programs have been designed, but testing has not been completed yet.

Edge diffraction

The DRT programs described above allow dynamic ray tracing of edge-diffracted rays from the source to a receiver lying in the shadow of WKBJ rays of a specified kind. An edge-diffracted ray is traced by the method described in the previous section from the edge point to the receiver. At the edge an appropriate version of Snell's law is satisfied and the amplitude is multiplied by the appropriate diffraction coefficient (Hanyga, 1985).

For signal paths in the transition zone of a shadow boundary uniform asymptotic formulae involving Fresnel integrals are used (Achenbach et al, 1982; Hanyga, 1985).

Anisotropy and prestress

The references quoted in the two previous sections deal with arbitrary anisotropic and possibly prestressed linear elastic media. FORTRAN programs based on these references cover the cases of isotropic blocks, transversely isotropic blocks with fixed orientation and with variable orientation of isotropy axes. Detailed formulae for transversely isotropic media are taken from Hanyga, unpublished.

Kinematic ray tracing in 3D

Preliminary to DRT a ray with a prescribed ray code is traced from the point source to the receiver. The associated two-point boundary value problems are solved in the following steps.

Step 1. A large collection of piecewise straight rays is traced through a number of more important interfaces. These include all the efficient reflectors in the model. At each of these interfaces the ray splits into transmitted, equal-angle reflected and two converted reflected rays (the latter are not traced by some programs). The rays are intercepted at their intersection with a surface, a sphere or a cylinder and stored on files.

Step 2. The rays from the above files are used as approximate rays in iterative programs tracing rays through homogeneous isotropic blocks. Polak's algorithm (Polak, 1974) is used. Edge-diffracted rays are traced by the same program.

Step 3. The result of the iterative procedure of Step 2 is fed into a program tracing rays through inhomogeneous and possibly anisotropic blocks. The algorithm is based on the idea of parallel shooting and tracing in fictitious time (Hanyga, 1984a) and uses Polak's version of the method of secants (Polak cit.).

The iterative program through isotropic homogeneous blocks takes 20-30 seconds in the SATM2 models. The programs of Step 3 take a few minutes to run.

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