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VII.8 A new technique for 3-component seismogram analysis

The individual seismograph stations of global networks always include three-component instrumentation for the very simple reason that the seismic wavefield comprises both vertical and horizontal ground motions. Seismologists have for may years successfully exploited the information potential of three-component records for wave propagation modelling, retrieval of structural information (imaging) ans source parameters, but these efforts have mainly been confined to the low frequency (less than 0.2 Hz) part of the wavefield. Much effort has been invested in extracting similar information in the high frequency band, say 1-10 Hz, but efforts generally have met with little success. The reason for this appears to be twofold:

1) High frequency records are rather complex due to scattering, mode conversions and multipathing.
2) The techniques of analysis often fail to produce wavefield parameters in an easily interpretable format. For example, a common procedure is to produce many particle-motion plots which generally are difficult to interpret.

In this section we present a new approach for extracting wavefield parameters, based on *a priori* models for P and S wave particle motions. Special attention has been given to the problem of presenting results in an easily interpretable manner for extracting signal parameters convenient for a wide variety of research applications. This has provided some remarkable results using data from the NORESS array, which were presented in the previous Semiannual Technical Summary (Christoffersson et al, 1985).

Structure of 3-component registrations

Both P and S waves exhibit a high degree of linear polarization. Ground particle motion coincides with the azimuth of propagation for the P phases. Surface waves of the Rayleigh type are generally elliptically polarized in the vertical-radial plane, the fundamental modes displaying retrograde particle motion and the higher modes prograde ellipticity. Surface Love waves are also found to be rectilinearly polarized, but in a horizontal plane orthogonal to the direction of wave propagation. Microseismic background is of the Rayleigh type, but with little preferred directionality. Signal-generated noise may also be polarized, although the direction of polarization is often random in nature. Using these various characteristics of polarized particle motion trajectories, we proceed in the following way:

Notations:

$$y(t) = [y_1(t),\ y_2(t),\ y_3(t)]^*$$

is the observed 3-component data vector where y_1 is the north component, y_2 the east and y_3 the vertical. The asterisk denotes the transpose.

$$z(t) = [z_1(t),\ z_2(t),\ \ldots\ z_k(t)]^*$$

is a k-dimensioned representation of the signal.

$$\Lambda = \begin{pmatrix} \lambda_{11} & \cdots & \lambda_{1k} \\ \vdots & & \\ \lambda_{31} & & \lambda_{3k} \end{pmatrix}$$

is a matrix of unknown constants relating the signal to the observed 3-component data.

$$\varepsilon(t) = [\varepsilon_1(t),\ \varepsilon_2(t),\ \varepsilon_3(t)]^*$$

is the noise vector.

Model

We assume that the observed data vector y(t) allows the following representation

$$y(t) = \Lambda(t) \rightarrow \varepsilon(t) \qquad (1)$$

with the following basic assumptions:

i) z(t) and ε(t) are orthogonal (uncorrelated)

ii) ε(t) has expectation zero

iii) the components of z(t) are linearly independent, i.e., the signal is k-dimensional

iv) all moment up to at least second order exist.

From these assumptions it follows that the zero-lag second order moments of y(t) can be written

$$\Sigma(t) = \Lambda\ \phi(t)\ \Lambda^* + \Psi(t) \qquad (2)$$

where $\Sigma(t) = E\left[y(t)\ y(t)^*\right]$
$\phi(t) = E\left[z(t)\ z(t)^*\right]$
$\Psi(t) = E\left[\varepsilon(t)\ \varepsilon(t)^*\right]$

Integrating (or averaging in discrete time) eq. (2) over a time window $T_0 < t < T$ we obtain the following, second-order structure

$$\Sigma = \Lambda\ \phi\ \Lambda^* + \Psi \qquad (3)$$

$$\Sigma = \int_{T_0}^{T} \Sigma(t)\ dt = \left[\frac{1}{T-T_0+1} \sum_{t=T_0}^{T} \Sigma(t)\right]$$

$$\phi = \int_{T_0}^{T} \phi(t)\ dt = \left[\frac{1}{T-T_0+1} \sum_{t=T_0}^{T} \phi(t)\right]$$

$$\Psi = \int_{T_0}^{T} \Psi(t)\ dt = \left[\frac{1}{T-T_0+1} \sum_{t=T_0}^{T} \Psi(t)\right]$$

Eq. (3) is the ordinary factor analysis model.

Before discusisng the estimation of the unknown parameters, usually based on the second order measurements, some details on the P and S wave representations will be given - at this time the surface wave representations are excluded.

<u>Body wave representations</u>

For P-waves the signal is 1-dimensional and will be written as:

$$
\begin{aligned}
y_1(t) &= \lambda_{11}\ z_1(t) + \varepsilon_1(t) \\
y_2(t) &= \lambda_{21}\ z_1(t) + \varepsilon_2(t) \\
y_3(t) &= \lambda_{31}\ z_1(t) + \varepsilon_3(t)
\end{aligned}
\tag{4}
$$

In terms of the second order structure we have

$$\Lambda = \begin{Bmatrix} \lambda_{11} \\ \lambda_{21} \\ \lambda_{31} \end{Bmatrix} \qquad \phi = \text{signal power} = (1 \times 1 \text{ matrix})$$

$\Psi =$ covariance matrix of the noise.

For P-waves the second order moments would be:

$$\Sigma_o = \Lambda_p\ \Phi_p\ \Lambda_p{*} + \Psi \tag{5}$$

where (given rotated in z, T, R)

$$\Lambda_p = \begin{Bmatrix} \lambda_{11} \\ 0 \\ \lambda_{31} \end{Bmatrix} \text{ and } \quad \phi_o = (\phi_{11})$$

and $\mathrm{sgn}(\lambda_{11}) \neq \mathrm{sgn}(\lambda_{31})$.

In the case of S-waves the signal is 2-dimensional, with SH and SV components.

$$
\begin{aligned}
y_1(t) &= \lambda_{11}\, z_1(t) + \lambda_{12}\, z_2(t) + \varepsilon_1(t)\\
y_2(t) &= \lambda_{21}\, z_1(t) + \lambda_{22}\, z_2(t) + \varepsilon_2(t)\\
y_3(t) &= 0 \cdot z_1(t) + \lambda_{32}\, z_2(t) + \varepsilon_3(t)
\end{aligned} \tag{6}
$$

where $z_1(t)$ is the SH component and $z_2(t)$ the SV component.

In terms of the second order structure we have

$$
\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ 0 & \lambda_{32} \end{bmatrix} \quad \text{and} \quad \phi = \begin{bmatrix} \text{power of SH} & 0 \\ 0 & \text{power of SV} \end{bmatrix}
$$

with the columns of z being orthogonal.

In this case, the second order moments would be:

$$
\Sigma_s = \Lambda_s\, \phi_s\, \Lambda_s{*} + \Psi \tag{7}
$$

where (given rotated z, T, R)

$$
\Lambda_s = \begin{pmatrix} 0 & \lambda_{12} \\ \lambda_{21} & 0 \\ 0 & \lambda_{32} \end{pmatrix} \quad \text{and} \quad \phi_s = \begin{pmatrix} \phi_{11} & 0 \\ 0 & \phi_{22} \end{pmatrix}
$$

and $\mathrm{sgn}(\lambda_{12}) = \mathrm{sgn}(\lambda_{32})$.

The first column in Λ_s is the SH component with signal power ϕ_{11}, the second is the SV component with signal power ϕ_{22}.

Estimation

For a 3-component station, the Σ's in eqs. (5) and (7) are symmetric 3x3 matrices having at most 6 different elements. In order to make the covariance structures identified, we can have at most

6 unknown parameters in Λ, ϕ and Ψ, so we must introduce some restrictions. Firstly, we will normalize the non-zero element in the ϕ-matrices to be equal to 1 which is actually not a restriction. Secondly, we assume that the Ψ-matrix is diagonal, i.e., the noise is assumed to have no zero-lag correlation. Further, in some cases we will use the additional restriction that Ψ is proportional to some known diagonal matrix Ψ_0, i.e., $\Psi = \theta\ \Psi_0$.

The estimation of the unknown parameters will be based on the observed second order moment of the observations over a time window

$$S = \frac{1}{T-T_0+1} \sum_{t=T_0}^{T} y(t)\ t(t)^* \tag{8}$$

There are several possible estimators that can be used. Most are based on some fitting function, i.e., minimizing the difference between the observed S and the theoretical Σ.

The most commonly used are

i) ML (MAXIMUM LIKELIHOOD)

This estimator is derived from Gaussian assumptions and minimizes:

$$F = \log |\Sigma| + \mathrm{tr}(S\Sigma^{-1}) - \log |S| - q \tag{9}$$

where q is the dimension of the data (in this case q=3), and tr = the trace operator.

ii) ULS (UNWEIGHTED LEAST SQUARES)

This estimator minimizes:

$$F = tr(S-\Sigma)^2 \tag{10}$$

i.e., the sum of squares of all the elements in $S-\Sigma$.

iii) GLS (GENERALIZED LEAST SQUARES)

This estimator minimizes:

$$F = tr(I-S^{-1}\Sigma)^2 \tag{11}$$

iv) PC (PRINCIPAL COMPONENT)

This estimator maximizes, subject to normalizing conditions on Λ:

$$F = tr(\Lambda^* S \Lambda) \tag{12}$$

Properties of estimators

The PC estimator leads to an eigenvalue decomposition of the observed moment matrix S and was suggested by Husebye et al (1967). It differs from the other three mainly in that it focuses heavily on the diagonal of S whereas the other put more weight on the off-diagonal. Under Gaussian's assumption, the ML and GLS estimates are asymptotically equivalent and it is possible to obtain a statistical test for the model fit to data. ML and GLS need a positive definite S-matrix, whereas ULS works even with non-grammian S. This property would point in favor of ULS because for strong and clear P-waves much above ground noise, the observed S-matrix will often be near singular. However, this problem can be overcome by attenuating the transverse component, i.e., by adding white noise. We will therefore use the ML estimator, mainly because of its possibility for model tests. These are of course strictly valid only under the Gaussian assumptions. However, as has been demonstrated in the applications, the tests can be modified to work well in practice.

Model test

Under the assumption of independent observations and Gaussian distribution, model tests can be carried out based on the ML estimator. We have approximately that

$$(N^{+}-1) \cdot F \quad \text{with } N^{+} = (T-T_0+1) \tag{13}$$

is distributed as chi-square with degrees of freedom equal to $q(q+1)/2$ - number of unknown parameters. Test for the P-wave can then be constructed in the following way:

We assume that $\Psi = \phi^2 I$. Let F_N and F_P denote the minimum of the fitting functions for the Noise and P models, respectively, using ML.

Then $(N^{+}-1)F_N$ is approximately chi-square with 5 d.f., $(N^{+}-1)F_P$ approximately chi-square with 3 d.f., and the difference $(N^{+}-1)(F_N-F_P)$ is chi-square with 2 d.f. Now, if data really are noise with $\Psi = \phi^2 I$, both the Noise and the P model would have reasonable fit to data and the drop in chi-square, i.e., $(N^{+}-1)(F_N-F_P)$ would not be significant.

For data generated by a P-wave, on the other hand, the noise model would have poor fit, the drop in chi-square would be significant and the P model would have good fit to data.

Define

$$
\begin{aligned}
P(P) &= \text{Probability} \quad \chi^2(3) > (N^*-1)\, F_P \\
P(N) &= \quad '' \quad \chi^2(5) > (N^*-1)\, F_N \\
P(D) &= \quad '' \quad \chi^2(2) > (N^*-1)\, (F_P-F_N)
\end{aligned} \tag{14}
$$

We then use

$$P(P) \cdot P(PE>0) = P(P)(1-P(D)) \tag{15}$$

i.e., the "probability of a P-wave multiplied with the probability of the P energy being larger than zero".

This measure is rather sensitive to deviation from the Gaussian and independent assumptions. Also, for large N it is sensitive in the sense that small deviations from the P-wave model will easily be detected. This problem can be solved by multiplying N^+ with a constant $C(0<C<1)$, the size of which depends on the sampling rate and frequency. The actual value of C can be determined such that $P(P) \cdot P(PE>0)$ is small for noise data and large for data containing P signals. Similar tests can easily be determined for S-, R- and L-waves.

Concluding remarks

The 3-component analysis techniques as described above have been tested on a large variety of seismic recordings and a few examples here will be mentioned. For the broadband records of the recent large Mexican earthquake (unfiltered) at HRV (Harvard, Mass., USA) we easily identified phase arrivals like P, PP, PPP, PcP, ScP and S. The corresponding azimuth estimates deviated only around ± 2 deg from the true ones - thus, from a single 3-component station record, a good initial epicenter estimate is feasible. Very many NORESS records have also been analyzed, and for teleseismic events adequate estimates of the slowness-vector are obtainable. In the extreme, deterministic scattering contributions in the P-wave coda have been identified. Our 3-component analysis techniques also work quite well, particularly for P-wave motion, for vertical seismic profiling (VSP) records where dominant signal frequencies typically are 50-100 Hz.

Another feature of this technique is tht it works quite well for weak signals, which is not exactly surprising as here we exploit the wavefield structure in contrast to stacking techniques where signal power variations over the earth's surface are used.
One drawback with our 3-comp. analysis is pure noise triggering, which is attributed to some P-wave energy in the background noise; the so-called whispering mantle effect. Therefore, in an on-line event detection context 3-comp. triggering has to be weighted by signal power estimates say on the vertical component before event presence is declared.

Finally, we consider that 3-comp. analysis should be potentially very useful in an "expert system" context (e.g., see Chen, 1983), as in this way a systematic wavefield decomposition is feasible even at relatively low SNRs, whereas conventional "signal power" techniques like spectral and f-k analysis become inefficient.

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