NORSAR

ROYAL NORWEGIAN COUNCIL FOR SCIENTIFIC AND INDUSTRIAL RESEARCH

NORSAR Scientific Report No. 1-85/86

FINAL TECHNICAL SUMMARY

1 April - 30 September 1985

L. B. Loughran (ed.)

Kjeller, December 1985

VII.6 The calculation of ellipticity corrections

For many purposes, including that of evaluating the performance of GDSN stations in source analysis studies, it is necessary to calculate synthetics, or Green's functions, in a spherically symmetric earth model (or a combination of such models), but with the effects of asphericity of structure included to first order. the effects are of primary significance to the travel times of waves, and the most conspicuous such effect is that due to ellipticity. Since the ellipticity correction is to be applied routinely to a variety of phases and for many source-receiver combinations, it is important that an effective computational method is available. Dziewonski and Gilbert (1976) obtained expressions for the ellipticity correction in the form of an integral over epicentral distance. On the other hand, the spherical travel times are usually evaluated much more efficiently as an integral over radius. First order perturbations in these travel times can be evaluated similarly; in fact, the ellipticity correction can be obtained as a by-product of the spherical travel time calculation.

The one-way spherical travel time of a ray between levels r_0 and r_1 can be expressed:

$$T = \int_{r_0}^{r_1} \frac{d(\eta^2 - p^2)^{\frac{1}{2}}}{1 - \eta\, v'} \qquad (1)$$

where p is the ray parameter, $\eta = r/v$, and $v' = dv/dr$ in the spherical reference model. For a turning ray, r_0 is the turning point radius. Following Dziewonski and Gilbert (1976), if the asphericity is given by

$$\delta v(r,\theta,\phi) = -v'(r)\ \delta r(r,\theta,\phi)$$

and letting

$$\delta r(r,\theta,\phi)/r = \sigma(r,\theta,\phi)$$

the perturbation in T can be expressed:

$$\delta T = \int_{r_0}^{r_1} \eta\, v^1\, \sigma \frac{d(\eta^2-p^2)^{\frac{1}{2}}}{1 - \eta\, v'} \pm \left[\sigma(\eta^2-p^2)^{\frac{1}{2}}\right]_{r_0}^{r_1} \qquad (2)$$

where in the last term, +/- is to be taken for down-/upgoing waves. For a complete ray the last term is always cancelled by contributions from subsequent layers, unless r_0 or r_1 represents a velocity discontinuity or an end-point of the ray.

Numerically, T in eq. (1) is evaluated by interpolating v between r_0 and r_1. For example, for $v = a(r/r_c)^b$:

$$T = \frac{1}{1-b} \left[(\eta^2-p^2)^{\frac{1}{2}}\right]_{r_0}^{r_1} \qquad (3)$$

Similarly, if we choose (r_1-r_0) sufficiently small so that σ can be averaged:

$$\delta T = \left(\frac{1}{2} \frac{b}{1-b} \pm 1\right) \left[\sigma(\eta^2-p^2)^{\frac{1}{2}}\right]_{r_0}^{r_1} \qquad (4)$$

To substitute for σ we need the lateral position of the ray at r_1 and r_0. This information can be obtained from the change in epicentral distance:

$$\Delta = \pm \frac{1}{1-b} \left[\cos^{-1}(p/\eta)\right]_{r_0}^{r_1} \tag{5}$$

Further simplifications are possible if σ represents the ellipticity. As a check on the accuracy of the calculations we compared the results to tabulated results (Dziewonski and Gilbert, 1976), and Fig. VII.6.1 shows an example for surface focus S waves. Considering the difference in reference model and possibly in the values for ellipticity (we used the values of Bullen, 1975), the agreement is quite satisfactory.

D.J. Doornbos

References

Bullen, K.E. (1975): The Earth's density. Chapman and Hall Ltd., London, pp. 420.

Dziewonski, A.M. and Gilbert, F. (1976): The effect of small, aspherical perturbations on travel times and a re-examination of the corrections for ellipticity. Geophys. J. R. Astron. Soc., 44, 7-17.

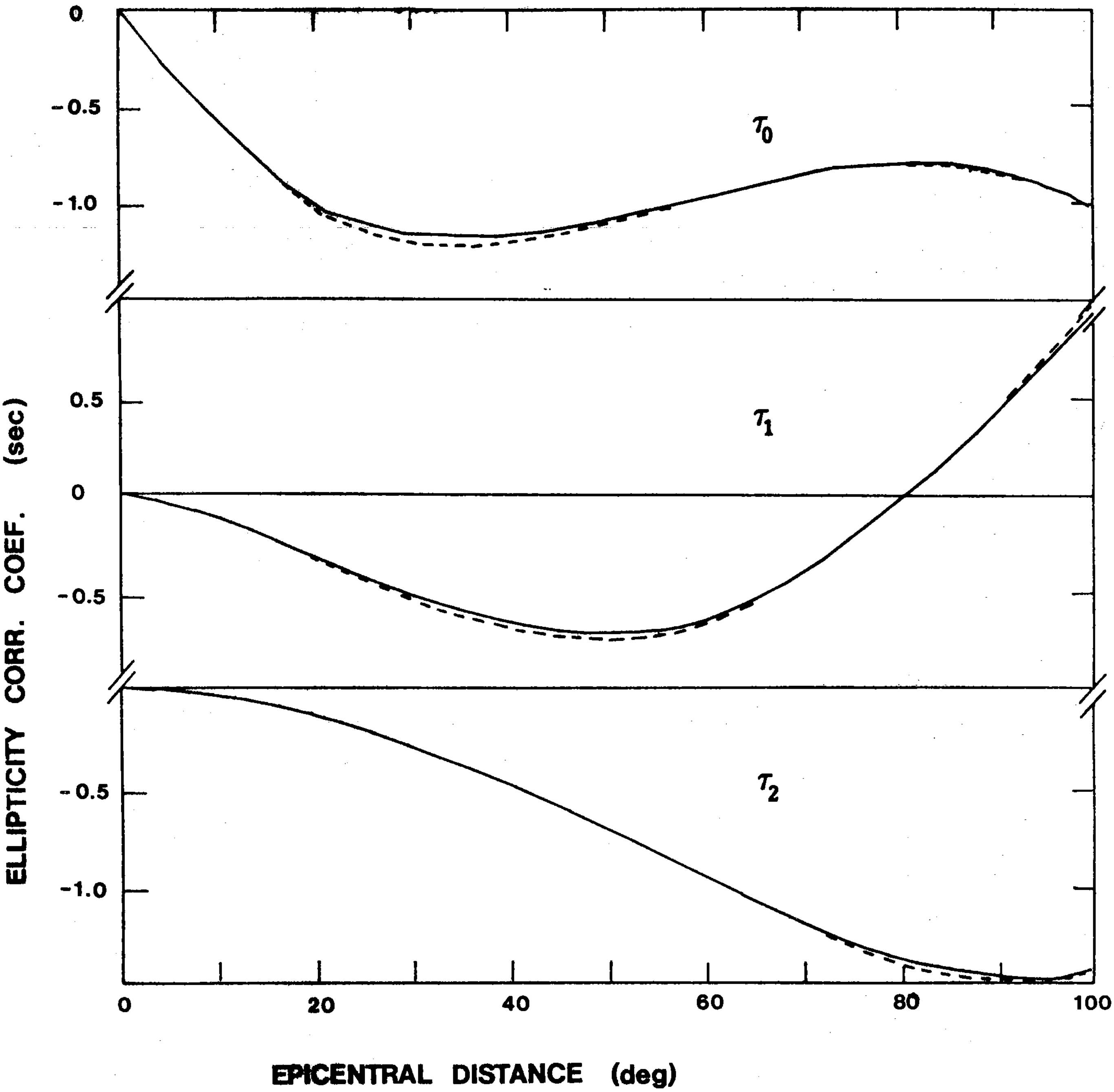

Fig. VII.6.1 Comparison of ellipticity correction coefficients for surface focus S phase
————: this study; ---------: Dziewonski and Gilbert (1976).