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VII. 1 A scattering model of regional $P_{n}$ wave propagation

The $P_{n}$ phase is crucial for the detection and location of regional events. It is therefore important to determine its characteristic properties. The properties of $P_{n}$ are well understood in onedimensional crust-mantle models: $P_{n}$ is an ordinary head wave associated with the Moho in models of uniform plane layers and $P_{n}$ can be interpreted as a sum of whispering gallery waves forming a so-called interference head wave in models of spherical layers and/or models having a positive velocity gradient below the Moho (Menke and Richards, 1980). The phenomenon of the whispering gallery can explain the relatively long duration and high frequencies of $P_{n}$ at teleseismic distances. However, it cannot explain the of ten observed relatively large amplitudes in the later part of the signal. Similar characteristics are observed at regional distances. To determine the cause of these characteristics we have analyzed in some detail the NORESS records of $P_{n}$ from a suite of 6 mining explosions in the Blåsjø area. The explosion site is about 300 km from NORESS in an azimuth direction of $240^{\circ}$. A typical record section of $P_{n}$ from these explosions is shown in Fig. VII.I.1.

It can be seen that the first arrival is relatively weak. In fact, this arrival is easily missed for small events, whence $P_{n}$ is often associated with the dominant part of the wave train. We have applied wide-band $f-k$ analysis (Kværna and Doornbos, 1986) to the $P_{n}$ wave trains from all 6 events. Slowness solutions as a function of frequency are summarized in the velocity-azimuth spectrum of Fig. VII.1.2. The solutions show a pronounced and consistent anomaly within the range $2-4 \mathrm{~Hz}$; this is also the range where the signal has its maximum energy (Fig. VII.1.3). From an analysis of slowness as a function of time (Fig. VII. 1.4 displays the results for one of the events)
we infer that the first arrival has a slowness and azimuth consistent with $P_{n}$ in a one-dimensional crust-mantle model, and the anomaly is related to wave energy being delayed by about $0.5-0.6$ seconds. The anomaly cannot be generated locally near the surface since this would significantly perturb the wave front; the wavenumber solution shows that the wave front is in fact very nearly plane. The observational results suggest that these " $\mathrm{P}_{\mathrm{n}}$ " waves are actually the result of scattering at depth. Ray tracing backward in the direction given by the measured slowness leads to the result that the $0.5-0.6 \mathrm{~s}$ time delay is explained by a scattering source within the depth range $30-40 \mathrm{~km}$. The Moho here is usually taken to be at about 35 km depth. Hence the observational results are consistent with scattering by topographic relief of the Moho. An interesting geological aspect is that the inferred location of the proposed topographic feature, 70 km from NORESS in the southwest direction, coincides with the border of the Oslo Graben.

Scattering will of course affect all waves interacting with a rough Moho discontinuity. However, the scattered waves usually arrive in the coda of a relatively strong primary wave. In contrast, the first arriving $P_{n}$ is relatively weak due to the small coefficient of refraction through the Moho, and scattering due to topography of the boundary may dominate the wave train.

To illustrate these concepts, we can apply a recently developed method for modelling scattering by topographic relief (Doornbos, 1988). The first applications of this method concerned the relatively complicated case of a solid-liquid boundary. We have done some numerical experiments with a liquid layer over a solid half space, letting velocities and densities correspond to values in the crust and mantle. One of the results is given in Fig. VII.1.5. Here we have assumed a boundary
topography characterized by a correlation length of 6 km and an average height of 1 km , and the wave field is taken monochromatic at 2 Hz . The figure shows energy flux of P transmitted upward through the boundary, as a function of slowness of incident $P$. Two modes of scattering are shown: (1) The specular flux $E^{0}$ in the direction defined by the plane wave-plane interface concept. The specular flux through a rough interface is reduced with respect to the flux through a plane. (2) The diffuse flux $E S C$ due to multiple scattering in all (upward) directions. $E^{S C}$ does not exist for a plane interface. The figure illustrates well the sharp increase of the ratio $E S C / E O$ as the slowness approaches the critical value corresponding to $P_{n}$, thus supporting in a qualitative way the scattering model for propagation of this wave.

It should be noted that $E^{\circ}$ and $E S C$ are not observable parameters. What can be inferred is the flux at a receiver location on the surface. To model this we need to know the areal extent of topography, and the decay factor accounting for attenuation of $P_{n}$ along the Moho. Clearly these parameters as well as those describing the topography needed to compute $E^{S C}$ and $E^{O}$ are presently not or only poorly constrained, and further experiments are needed to establish useful constraints. Our preliminary results nevertheless suggest that the scattering model of $P_{n}$ wave propagation is viable, and consequently that a careful calibration is needed before using this phase for event location and velocity determination purposes.
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## References

Doornbos, D.J. (1988): Multiple scattering by topographic relief with application to the core-mantle boundary. Geophys. J., in press.

Kværna, T. and D.J. Doornbos (1986): An integrated approach to slowness analysis with arrays and three-component stations. Semiannual Technical Summary, 1 Oct 1985 - 31 Mar 1986, NORSAR Sci. Rep. No. 2-85/86, Kjeller, Norway.

Menke, W.H. and P.G. Richards (1980): Crust-mantle whispering gallery phases: A deterministic model of teleseismic $P_{n}$ wave propagation. J. Geophys. Res. 85, 5416-5422.


Fig. VII.1.1 Noress record of the $P_{n}$ phase from an explosion in the Blåsjø area. The top trace is the single channel observation, and the lower trace is the steered beam, both bandpass filtered between 2.0 Hz and 4.0 Hz . Note the relatively weak first arrival at about 2.0 seconds compared to the stronger signal arriving $0.5-0.6$ seconds later.

$$
\begin{gathered}
\text { SLOWNESS ANALYSIS - BLASJØ EVENTS } \\
\text { MOVING FREQUENCY BANDS - PN PHASE } \\
\text { BAND: } 0.8 \mathrm{HZ} \text { STEP: } 0.4 \mathrm{HZ}
\end{gathered}
$$




Fig. VII.1.2 Velocity and azimuth estimates for the $P_{n}$ phases as a function of frequency. The six events were processed by the wide-band method in 0.8 Hz wide frequency windows with 0.4 Hz steps. Between 2 and 4 Hz the deviation from theoretical azimuth is between 10 and 15 degrees, and the apparent velocity is significantly lower than expected for the $\mathrm{P}_{\mathrm{n}}$ phase.

BLAASJOE
MSAN UNCORRECTED SPECTRA


Fig. VII.1.3 This figures shows the uncorrected power density spectra of both the $P_{n}$ phase and the preceding noise for a typical Blåsjø event.


Fig. VII.1.4 Slowness estimates for one of the Blåjø events as a function of time. The data were processed by the wideband method in the frequency range $2.0-4.0 \mathrm{~Hz}$. The traced data were prealigned to compensate for offset across the array.


Slowness (s/km)

Fig. VII.1.5 Energy flux at 2 Hz through rough solid-liquid interface with the velocity-density structure $v_{p}+/-\frac{1}{3}=6.8 / 8.1 \mathrm{~km} / \mathrm{s}$, $\mathrm{v}_{\mathrm{s}}+/-=4.5 / 0 \mathrm{~km} / \mathrm{s}, \quad \rho^{+} /-=2.9 / 3.4 \mathrm{~g} / \mathrm{cm}^{3}$. The boundary roughness is characterized by an average height of 1 km and a correlation length of 6 km . The scattered flux is upward through the boundary. - specular direction; ................ : integrated diffuse flux.

