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7.2 A new ML scheme for 3-component slowness estimation which incorporates the crustal transfer function

Background

Efforts to exploit the wavefield information contained in the records of 3-component (3C) seismometers, say in terms of polarization characteristics, slowness vectors and other types of signal attributes, have until recently been less successful in the high frequency band 1-100 Hz. This problem has to a large extent been overcome by introducing flexible particle motion models in combination with effective maximum likelihood (ML) schemes for signal parameter estimation from 3C recordings (Christoffersson et al, 1988; Roberts et al, 1989; and Roberts and Christoffersson, 1990). Their techniques allow us to produce quite precise automatic preliminary bulletins for single 3C (digital) stations (Ruud and Husebye, 1991). A potential flaw in the above and similar techniques is that the response of the crust beneath the receiver is not incorporated in the signal model. However, recently Kennett (1991) suggested a new approach for decomposing the 3C seismograms of P, SV and SH waves incident onto the free surface by assuming that the slowness vector and the layered crustal structure beneath the station are known. A similar method can in principle be used for solving the inverse problem, and this is the subject dealt with here.

Method

The problem is that of estimating the azimuth φ and horizontal slowness p of the plane P-wave incident onto the layered structure below the free surface. In mathematical form, the model of our 3C recordings can be written as:

$$\bar{u}(\omega) = f(\omega_j)\bar{h}(\omega_j, \bar{s}, \bar{m}) + \bar{n}(\omega_j) \tag{1}$$

where $\bar{u}(\omega_j) = [u_N, u_E, u_Z]^T$ is the vector of discrete Fourier amplitudes at angular frequency ω_j for the individual recording components (NS, EW, Z) calculated for a time window containing the signal; $f(\omega_j)$ is the product of the incoming signal amplitude and the seismometer frequency response; $\bar{h}(\omega_j, \bar{s}, \bar{m}) = (h_N, h_E, h_Z)^T$ is the crustal transfer function for an incoming P-wave at frequency $f(\omega_j)$ for slowness vector $\bar{s}(\varphi, p)$, and the presumed known crustal structure given by the model vector $\bar{m}$; $\bar{n}(\omega_j) = (n_N, n_E, n_Z)^T$ is the noise vector and T means transposed. Assuming the noise is Gaussian and uncorrelated at different frequencies, then $E(\bar{n}_k, \bar{n}_j^{\,*}) = \delta_{kj}C_j$ where E is mathematical expectation, the asterisk * implies Hermitian conjugation and δ_{jk} is the Kronecker delta symbol. With the above assumption it can be shown that the maximum likelihood (ML) functional will be

$$L = 1/2\sum_j \ln(\det C_j) - 1/2\sum_j (\bar{u}_j - f_j\bar{h}_j)^* C_j^{-1}(\bar{u}_j - f_j\bar{h}_j) \tag{2}$$

where the summation is over N discrete frequencies in the selected signal bandwidth. L depends not only on the useful (informational) unknown parameters φ and p of the slowness vector, but also on the unknown regressor parameters $f(\omega_j)$. The ML estimates derived by maximization of the L functional (2) with respect to all unknown parameters will for a constant number of nuisance parameters $f(\omega_j)$ but an increasing sample size be asymptotically optimal estimates of the useful slowness vector parameters (Kushnir and Lokshtanov, 1988).

Presuming $\bar{s}(\varphi, p)$ is fixed, $\bar{m}$ is known a priori and that the noise is orthogonal between components and with equal power at all components and frequencies, i.e., $C_j = \sigma^2 I$ (I is the identity matrix), the maximum of L with respect to f_j is achieved at the points

$$f_j = \bar{h}_j^* \bar{u}_j / \bar{h}_j^* \bar{h}_j \tag{3}$$

By substituting the f_j here into (2) we have that the ML estimates of the informational parameters can be obtained by minimizing the following functional

$$\Theta(\varphi, p) = \sum_j \left\| \bar{u}_j - \bar{h}_j (\bar{h}_j^* \bar{u}_j) / \bar{h}_j^* \bar{h}_j \right\|^2 \tag{4}$$

Now the functional (4) depends only on the observations $\bar{u}_j$ and the informational parameters φ, p.

By using normalized transfer functions in the formula (4), i.e., $\bar{g}_j = \bar{h}_j / (\bar{h}_j^* \bar{h}_j)^{1/2}$ we obtain

$$\Theta(\varphi, p) = \sum_j [\bar{u}_j^* \bar{u}_j - (\bar{g}_j^* \bar{u}_j)^* (\bar{g}_j^* \bar{u}_j)] \tag{5}$$

Thus minimizing (4) is equivalent to maximizing the sum of squares of projections of our observational vectors $\bar{u}_j$ onto the theoretical directions determined by the model vectors $\bar{g}_j$.

The ratio of the projected to the observed energy in the selected frequency band

$$QL = \sum_j [(\bar{g}_j^* \bar{u}_j)^* (\bar{g}_j^* \bar{u}_j)] / \sum_j (\bar{u}_j^* \bar{u}) \tag{6}$$

can be used as a quality measure of our estimation. For example, if we have pure signal, $\bar{n}_j = 0$ for all j in eq. (1), then QL will be equal to 1. When $\bar{n}_j$ increases, QL will decrease.

The above slowness estimation technique is described in detail in a paper by Lokshtanov et al (1991), which also includes its extension to an array of M 3C stations presuming $\bar{s}_i$ and $\bar{n}_i$ to be identical.

Results -- velocity analysis of NORESS events

For the frequency band of interest for our analysis (1-15 Hz) the most important part of the crust would be the upper 5 km as conversion and/or reflections from deeper layers will be separated in time from the primary phase. To our knowledge the only available information on vertical velocity structures in this part of the crust comes from a study of Lokshtanov et al (1991), where upper crust velocity models were found by inversion of short period Rayleigh wave fundamental mode phase velocity measurements from NORSAR subarrays. Their results suggest an upper low velocity layer about 1 km thick or alternatively a two-layer model over a half space. Model parameters are given in Table 7.2.1

together with those for uniform velocity halfspace model. For comparison all three models have been used in the analysis of the event examples, and in the result presentation below they are referred to as the halfspace, one-layer and two-layer models.

Analysis procedure: The analysis was performed both for the central single 3C station and for the array of four 3C stations. After azimuth was determined, the maximum of the quality parameter QL and the corresponding apparent velocity was found by searching over a predefined slowness interval. The results are presented for a sliding time window with 20% cosine tapering and with an updating interval of 1/10 of the window length. The reference time for the estimated signal attributes is for the center of the time window.

Event 1 -- Teleseismic earthquake (Hindu Kush). This event, shown in Fig. 7.2.1, has a very high SNR and is dominated by relatively low frequency energy. The signal has an emergent onset and it starts at 9.2 s on the given time scale. The length of the sliding time window was 2 s and the analyzed frequencies are from 0.5 to 5.0 Hz. The QL-values rise as soon as the window moves into the signal and stay close to 1.0 well into the signal. Only for the velocity estimation do we find significant differences for the three models. The estimates for the layered models are quite stable and close to the expected velocity for an event at this distance (13.9 km/s). In contrast, the halfspace models give estimates biased towards high velocity -- static corrections can offset part of the bias. Regarding azimuth estimation, both the single station and the array estimates are biased relative to the expected azimuth of 96^{0} (about 10^{0} for the array and 15^{0} for single station).

Event 2 -- Local explosion (SE Norway). The window length is here 1 s and the analyzed frequency band is from 6 to 12 Hz. As seen from Fig. 7.2.2 the QL-values are also much lower for this event and in particular for the array where the signal correlation between stations is almost lost soon after the onset. For the three models a clear model dependency is seen in the velocity estimation for the single station. For the array it seems that the kinematic properties of the wavefield, i.e., phase shift between identical components at different stations, serves to stabilize the velocity estimates. This effect is hardly seen for the teleseismic event where the low signal frequencies result in much smaller phase shifts between stations. For the later part of the signal (after 10.0 s), the velocities are very high both for the array and the single station. By vespagram analysis of all the vertical NORESS sensors, we also find high velocities in this time interval (9-10 km/s) although not as high as for the 3C processing.

Discussion

For the uniform halfspace model the transfer function is frequency dependent and our estimation method becomes similar to previous techniques, e.g., Esmersoy et al (1986). Although the inclusion of layered structures clearly improved the estimated velocities, we do not obtain a corresponding improvement of the quality parameter QL which measures how well the observed data fit our signal model. The reason for this seems to be that the layered models affect mainly the ratio of the radial to the vertical amplitudes, while phase differences between components remain small -- everywhere within 30 from that of the halfspace. From the given examples and from general experience with the method it is seen that analysis of high frequency signals is relatively less successful than for low frequencies. An explanation here might be that the models used are not adequate at high frequencies -- being derived from Rg waves at periods from 0.7 to 1.7 s -- and/or that lateral variations are highly significant.

In conclusion, we have here demonstrated the importance of incorporating the crustal transfer in 3C analysis methods to ensure accurate estimates of P-wave slowness -- the technique will be extended to analysis of SV and SH waves.

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	Model	P-vel. (km/s)	S-vel. (km/s)	Density (g/cm^3)	Thickness (km)
1	1-layer	5.10	2.95	2.60	0.92
	halfspace	6.14	3.55	2.70	∞
2	2-layer	4.93	2.85	2.60	0.5
		5.40	3.12	2.60	0.5
	halfspace	6.12	3.54	2.70	∞
3	halfspace	5.9	3.14	2.60	∞

Table 7.2.1. The crustal models used in analysis. Model 1 and 2 are based on Rayleigh (Rg) dispersion analysis as reported by Lokshtanov et al (1991).

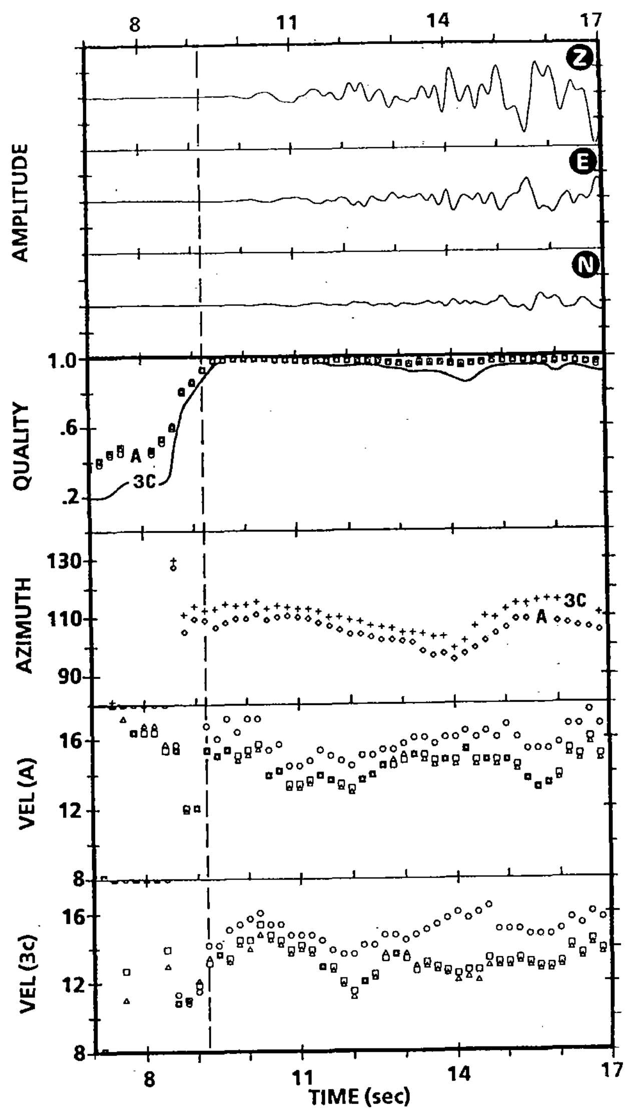

Fig. 7.2.1. Analysis results for Event 1 (Earthquake, Hindu Kush, July 29, 1985, origin time 07:54:44.5, backazimuth 95.7, distance 44.2). Processing parameters are given in text. From top and down the figure shows 3C seismograms (Z,E,N), quality parameter, azimuth, apparent velocity for the array of four 3C stations and apparent velocity for the single central 3C station. For quality and velocity the results for the three models are given with different symbols: circle (O) for halfspace, triangle (Δ) for one-layer and square (□) for the two-layer model. The model independent azimuth estimates are shown with plus (+) for single station and diamond (◊) for the 3C array. For quality the results for single 3C and 3C array are shown in the same diagram and are identified with "3C" and "A" respectively.

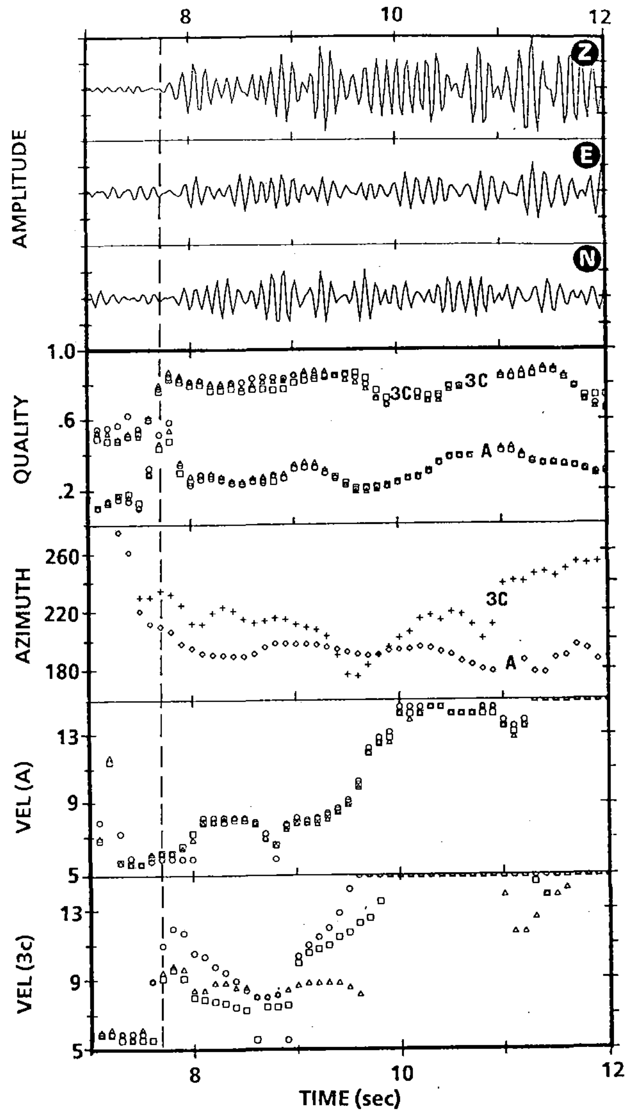

Fig. 7.2.2. Analysis results for Event 2 (Local explosion SE Norway, July 19, 1989, origin time approx. 11:17:50). Processing parameters are given in text. Figure layout and symbols as for Fig. 7.2.1.

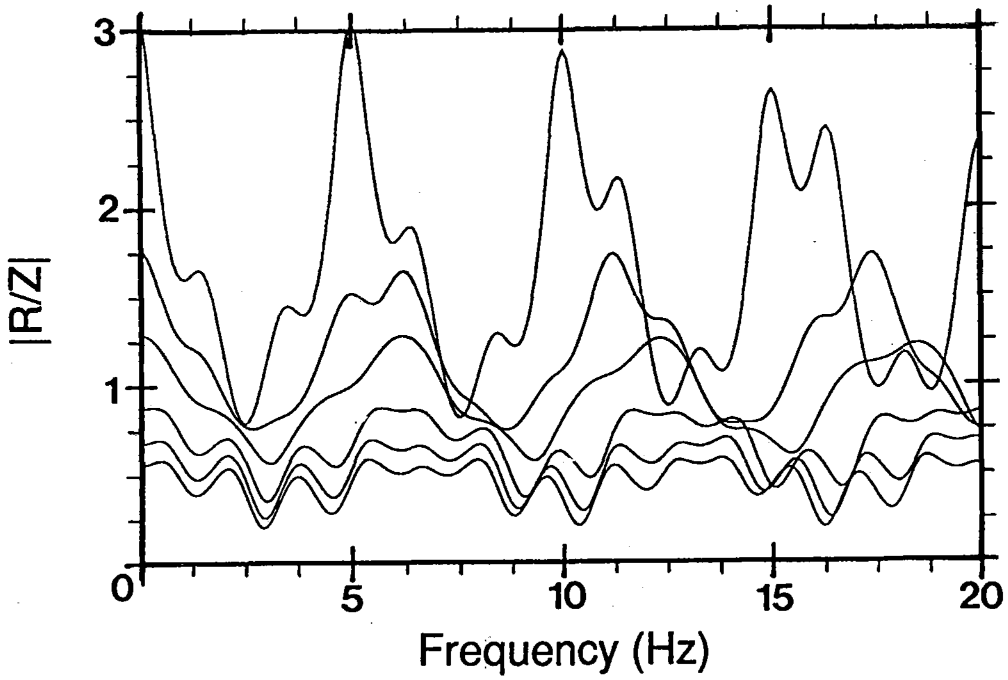

Fig. 7.2.3. Computed radial-to-vertical component amplitude ratio as a function of frequency for the one-layer over halfspace model in Table 7.2.1. Counted from the top of the six curves are for apparent velocities of 6, 7, 8 10, 12 and 14 km/s respectively. The corresponding phase shifts between components are within 30 from that of a halfspace model for all frequencies.The strong frequency dependence of the amplitude ratio illustrates the need for correcting velocity estimates for the effect of upper crustal layering.